Research During the Last Decade on Forced Convection Heat Transfer

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LECTURE

The aim of this lecture is to survey the progress in our understanding of convective heat transfer without phase changes which has been achieved since the last International Conference at London. This implies that the lecture has to cover a very wide area and that it is necessary to restrict it to the essential features in a way similar to surveying a landscape from an airplane where we can recognize only the large mountains and valleys, whereas detailed features remain undistinguishable. Many members of the heat transfer community have contributed to this progress and it will also not be possible to mention either all the names or all the important papers. The list of references should rather be taken as a source of more detailed information on the subjects touched on in this lecture.

A significant factor in convective heat transfer is the large number of boundary conditions encountered in various engineering applications. In our discussion, we will, accordingly, start from a standard situation for which heat transfer coefficients have been well established and we will investigate what variation in the heat transfer coefficients is produced by a variation in the boundary conditions. For boundary layer type flow, the selected standard is convective heat transfer caused by steady two-dimensional flow of a fluid with constant properties over a surface when the fluid is unlimited in extent, has a uniform velocity outside the boundary layer, and when the surface temperature is constant. This situation is usually referred to as flow over a flat plate.

The first question arising in this connection will be what the heat transfer is over an object of different geometry. In general, then, the velocity at the outside edge of the boundary layer will not be constant. This question has already been studied

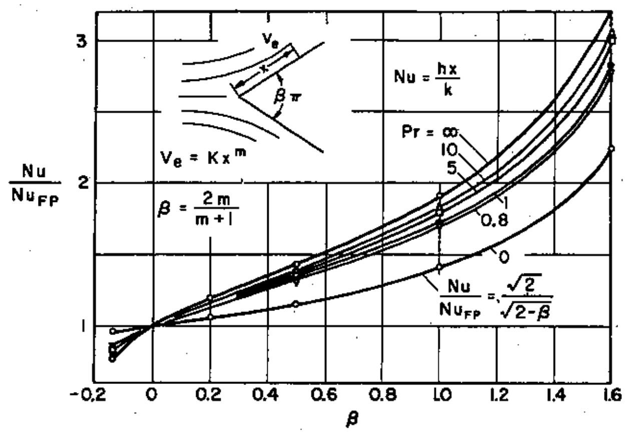

FIG. 1 — RATIO OF NUSSELT NUMBER FOR LAMINAR WEDGE FLOW TO NUSSELT NUMBER FOR FLAT PLATE AS FUNCTION OF WEDGE ANGLE PARAMETER.

prior to the last decade; it will, however, be included in the present discussion for the sake of completeness. Figure 1 considers laminar, steady flow of a constant property fluid over a family of geometries with constant surface temperature which produces a special type of velocity variation outside the boundary layer. The geometry consists of a wedge with various opening angles $\beta\pi$. The velocity V_e of inviscid flow along the surface of such wedges varies proportional to a power m of the distance x from the leading edge and the parameter β is connected with the exponent m by the relation given in the figure. For large Reynolds numbers, the velocity V_e of a viscous fluid has the same value outside the boundary layer. The ordinate of the figure displays the ratio of the actual local Nusselt number Nu to the Nusselt number Nu_{FP} on a flat plate for the same value of the

Reynolds number. The parameter β is used as abscissa of the main diagram and the Prandtl number appears as parameter. The data in the figure are from [1] and two limiting curves for $Pr = 0$ and $Pr = \infty$ have been added. It will be observed that the Nusselt number Nu is quite strongly dependent on the intensity of the velocity variation (β). It should, however, be pointed out that values of β larger than one are encountered rarely in engineering applications. A value $\beta = 1$ corresponds to two-dimensional flow in the neighborhood of a stagnation point, and a value of 0.5 to rotationally symmetric flow near a stagnation point.

An analysis of the special flow situation considered in Fig. 1 was comparatively easy because velocity profiles within the boundary layer are similar in shape for various locations x. This is not the case for flow over an object of arbitrary shape and the solution of the boundary layer equations becomes extremely time-consuming for such a situation. An approximation which is generally satisfactory for engineering calculations, however, may be obtained by the concept of "local similarity." This concept is based on the fact that a boundary layer fortunately has in general a poor memory for things which happened to it in the upstream region. Under the concept of local similarity, therefore, the boundary layer characteristics at a certain location are determined by accounting only for the factors influencing the boundary layer development at this location itself. Figure 2 compares the results of such an analysis (dashed line, [2]) with the results of measurements [3] for flow over a cylinder with circular cross section. The dashed line has been obtained specifically by selecting from the group of Nusselt numbers in Fig. 1 the one which exhibits at the same distance x from the stagnation point the same velocity V_e and the same velocity gradient dV_e/dx. The agreement between results obtained by the concept of local similarity and reality is generally found to be even better than in Fig. 2. This concept has more recently been applied with good success to hypersonic flow of air over blunt objects [4].

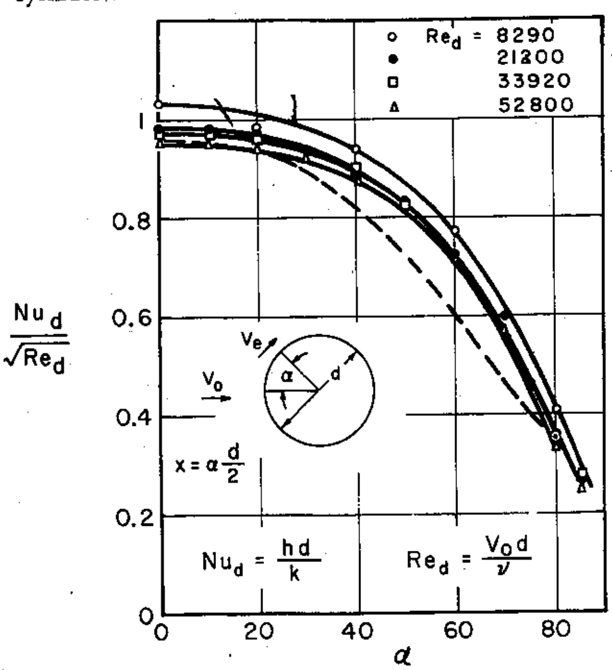

FIG. 2 – MEASURED LOCAL HEAT TRANSFER PARAMETER (FULL LINES) AND CALCULATION (DASHED LINE) BASED ON "LOCAL SIMILARITY" FOR FLOW OF AIR NORMAL TO A CIRCULAR CYLINDER.

Fortunately, a turbulent boundary layer has a memory for upstream occurrences which is even worse than that of a laminar one. Accordingly, it is generally sufficient for engineering calculations to apply the concept of local similarity by using as an approximation to the local heat transfer coefficient on the surface of an object with arbitrary shape the value which exists at the same location x on a flat plate for a velocity V_e which is equal to the velocity on the object under consideration at this location. The recourse to a family of similar flows is, therefore not required for a turbulent boundary layer.

A local variation of the surface temperature also influences heat transfer. Figure 3 demonstrates this in a diagram in which the ratio of the actual Nusselt number Nu to the Nusselt number Nu_{iso} on an isothermal surface is plotted over a parameter γ [5, 7]. Both the velocity variation along the outer border of the boundary layer (V_e) as well as the variation of the temperature difference θ between stream and surface are assumed to follow power laws as indicated in the figure. Again it should be noted that a large value of γ describes a wall temperature with a very steep temperature gradient at the location under consideration. It can be seen that the dependence of the heat transfer on the wall temperature variation is sufficiently small for a turbulent boundary layer that it can generally be neglected. It has, however, to be taken

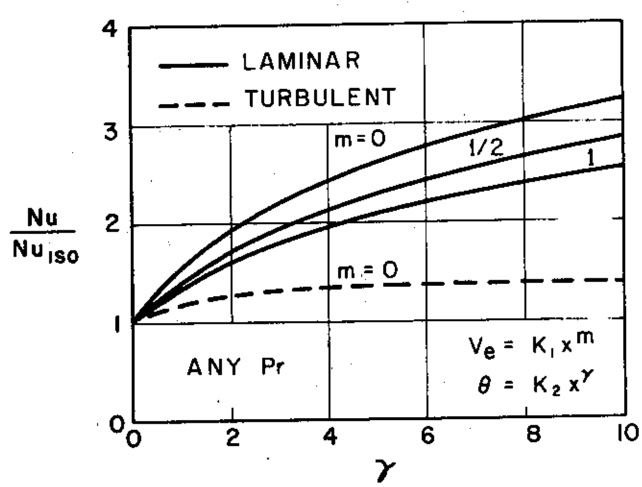

FIG. 3 – RATIO OF NUSSELT NUMBER FOR SURFACE WITH LOCALLY VARYING TEMPERATURE TO NUSSELT NUMBER FOR SURFACE WITH CONSTANT TEMPERATURE FOR WEDGE FLOW WITH LAMINAR AND TURBULENT BOUNDARY LAYER.

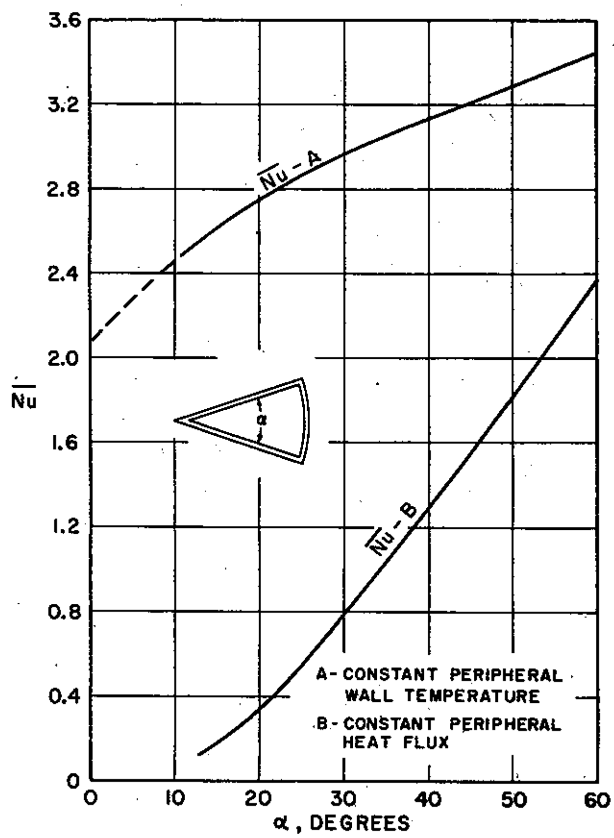

FIG. 4 – NUSSELT NUMBER BASED ON AVERAGE HEAT TRANSFER COEFFICIENT (AVERAGE HEAT FLUX PER UNIT AREA DIVIDED BY DIFFERENCE BETWEEN FLUID BULK TEMPERATURE AND AVERAGE WALL TEMPERATURE) AND HYDRAULIC DIAMETER FOR DEVELOPED LAMINAR FLOW OF AIR THROUGH A DUCT WITH CIRCULAR SECTOR CROSS-SECTION. WALL HEAT FLUX IS ASSUMED CONSTANT IN AXIAL DIRECTION.

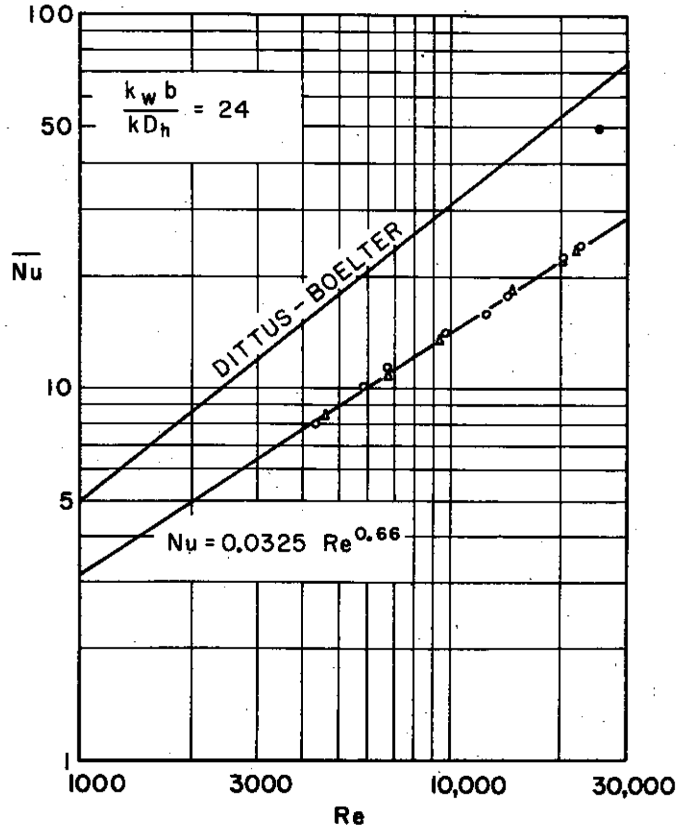

FIG. 5 – NUSSELT NUMBER BASED ON AVERAGE HEAT TRANSFER COEFFICIENT AND HYDRAULIC DIAMETER OBTAINED FROM EXPERIMENTS (OPEN SYMBOLS), FROM THE DITTUS-BOELTER RELATION FOR A TUBE WITH CIRCULAR CROSS-SECTION, AND FROM A NONCIRCULAR DUCT ANALYSIS (BLACK DOT) FOR DEVELOPED TURBULENT FLOW OF AIR THROUGH A DUCT WITH TRIANGULAR CROSS-SECTION.

k_w heat conductivity of duct wall, k heat conductivity of air, b wall thickness, D_h hydraulic diameter

into account for a laminar boundary layer. Fortunately, convenient procedures [6 to 8] have been worked out by which the influence of an arbitrary surface temperature variation can be calculated. The most versatile of those are based on the fact that, for a constant property fluid, the energy equation describing the temperature field is linear, and that therefore the law of superposition applies.

For duct flow, the dependence of Nusselt number on a wall temperature variation in axial direction is of the same order of magnitude as for boundary layer flow, and the same methods can also be used for the prediction of heat transfer. Another factor which has a very strong influence on heat transfer, however, has only been recognized in its importance in recent years. This is a variation of the wall surface temperature around the duct periphery. Such a variation is most frequently encountered in ducts with non-circular cross-section and its influence will be discussed in connection with the next two figures. Figure 4 [9] contains Nusselt numbers $\overline{Nu}$ averaged around the periphery of a duct with a cross-section having the shape of a circular sector with an opening angle α. Developed laminar flow and a developed temperature field of a constant property fluid flowing through the duct is assumed. Two curves are presented; the upper one holds for a peripherally uniform wall surface temperature. In a duct with non-circular cross-section, such a condition generates a heat flux which varies around the duct periphery and drops, for instance, in corners to a value zero. The lower curve indicates Nusselt numbers for a duct with constant peripheral heat flux from the duct surface into the fluid and, accordingly, with a surface temperature which varies around the circumference. It can be recognized that, for small opening angles α, the two Nusselt numbers may differ by an order of magnitude and that, even for an opening angle of 60 degrees, the difference is still approximately 50 per cent.

Figure 5 contains heat transfer coefficients, averaged over the circumference, for turbulent flow of air through a duct, the cross-section of which has the shape of an isosceles triangle with an opening angle of 12 degrees [10]. The open symbols indicate the results of measurements for developed flow and thermal conditions. A line is drawn through these points to obtain the empirical equation indicated in the figure. The upper curve presents the Nusselt number which is predicted from Dittus-Boelter's equation for a round tube with the same hydraulic diameter. The experimental conditions were such that one has to expect heat transfer coefficients approximately halfway between those for a constant peripheral wall temperature and a constant peripheral heat flux. It can be observed that the experimental heat transfer coefficients have only half the values predicted for a circular duct with the same hydraulic diameter. The influence of a circumferential temperature variation on heat transfer in duct flow is, therefore, very large even for turbulent flow, at least for cross-sections containing small angles. No method is presently available by which this effect can be predicted. The black dot in the figure is the result of an analytical calculation [10] for peripherally uniform heat flux and it can be recognized from the distance of this point from the line connecting the experimental data that the result is unsatisfactory for such an extreme duct shape. It appears that measurements on the detailed structure of the turbulent heat transport in such a duct are required as a basis of such an analysis. Figure 6 presents, as an example, turbulence intensities obtained by hot wire measurements in the same duct at a Reynolds number of 10,000. The lines in this figure connect locations with a certain ratio of the root mean square of the turbulent fluctuations u' in axial direction to the mean velocity u_m of the flow through the duct. More information on various turbulence correlations is contained in [11].

The analytical results presented up to now have been obtained for a constant property fluid and the experimental results were gained under conditions where the property variations are small. In many present-day engineering applications, on the other hand, very large temperature variations are encountered and the question arises how such property variations influence heat transfer and how they can be accounted for in engineering calculations. The

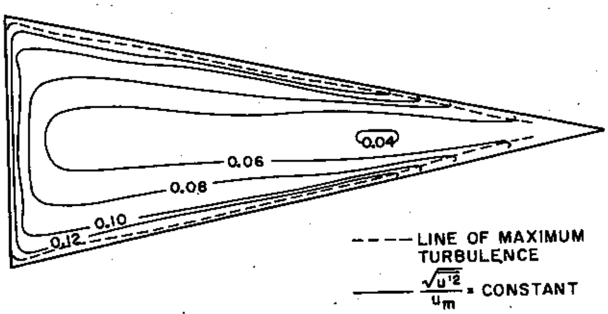

FIG. 6 – FIELD OF RATIO OF ROOT-MEAN-SQUARE VELOCITY FLUCTUATION IN AXIAL DIRECTION TO MEAN VELOCITY FOR TURBULENT FLOW OF AIR THROUGH A DUCT WITH TRIANGULAR CROSS-SECTION. Re = 10,900.

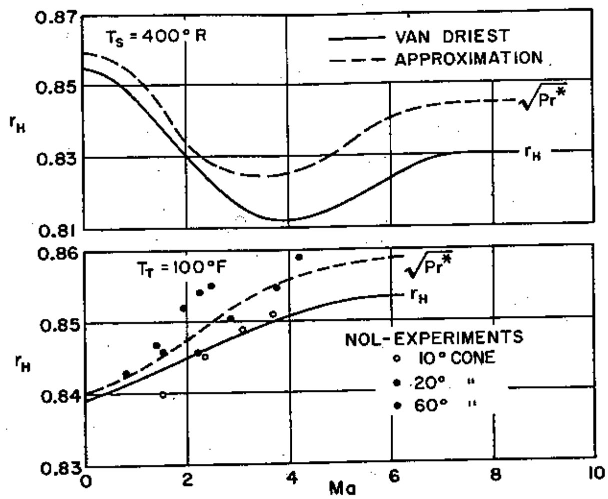

FIG. 7 – ENTHALPY RECOVERY FACTOR FOR LAMINAR SUPERSONIC FLOW OF AIR OVER A FLAT PLATE. THE RESULTS OF THE REFERENCE ENTHALPY METHOD (DASHED LINES) ARE COMPARED WITH EXACT BOUNDARY LAYER SOLUTIONS (FULL LINES). *Ma* Mach number.

following method has proved successful for gases and is in widespread use today: The equation defining the heat transfer coefficient is altered by introducing an enthalpy difference as driving potential instead of a temperature difference

$$q = h_H\,(H_r - H_w) \tag{1}$$

(q heat flow per unit area and time at wall surface, h_H heat transfer coefficient, H_r recovery enthalpy for high velocity flow or stream enthalpy outside boundary layer for low flow velocities, H_w enthalpy of gas at wall temperature.) Relationships for the recovery factor, for the friction factor, and for the Stanton number which hold for a constant property fluid are then used as an approximation for a variable property fluid as well and the properties are introduced into those relationships at a reference enthalpy which has to be determined empirically and which is found to be located within the enthalpy range limited by the extremes occurring in the specific problem. Reference [12] gives, for instance, for such a reference enthalpy H^* the relation

$$H^* = H_s + 0.5\,(H_w - H_s) + 0.22\,(H_r - H_s) \tag{2}$$

with H_s indicating the static enthalpy in the gas stream at the outer border of the boundary layer. Figures 7 and 8 [12] compare the results obtained by this procedure with the results of a boundary layer analysis by van Driest [13] for laminar flow of air with supersonic velocities over a flat plate with constant surface temperature. Figure 7 presents the enthalpy recovery factor

$$r_H = \frac{H_r - H_s}{V_e^2/2} \tag{3}$$

(V_e velocity on outer edge of boundary layer) which determines the plate surface temperature under zero

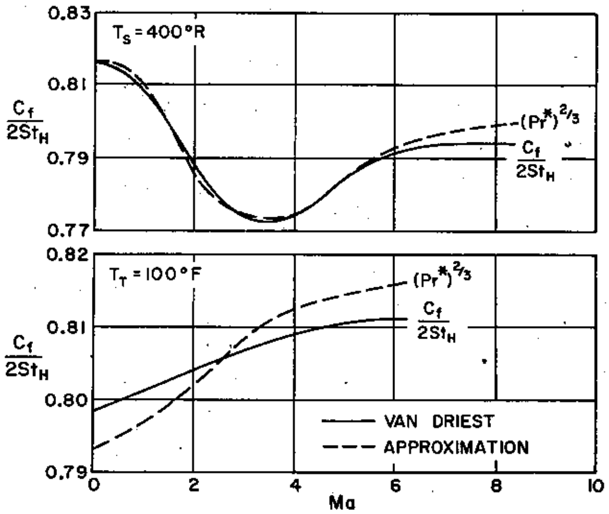

FIG. 8 – RATIO OF FRICTION FACTOR TO TWO TIMES THE STANTON NUMBER (BASED ON ENTHALPIES) FOR LAMINAR SUPERSONIC FLOW OF AIR OVER A FLAT PLATE. THE RESULTS OF THE REFERENCE ENTHALPY METHOD (DASHED LINES) ARE COMPARED WITH EXACT BOUNDARY LAYER SOLUTIONS (FULL LINES).

heat flux condition. For a constant property fluid, the recovery factor is equal to the square root of the Prandtl number. Figure 8 presents the ratio of the friction factor c_f to two times the Stanton number St_H which ratio assumes for a constant property fluid the value $Pr^{2/3}$. The agreement between the results of the simple approximate procedure (dashed lines) and the exact analysis (full lines) is, in both cases, within 2½ per cent. It was found that the agreement with the results of experiments for a turbulent boundary layer is also very good. A similarly satisfactory method for liquids has not yet been found and a relation covering all kinds of liquids may well not be possible. The equations (1) and (3) describing the heat flux at the wall surface and the recovery enthalpy can be combined in the following relation

$$q = h_H\left(H_s + r_H \frac{V_e^2}{2} - H_w\right) = h_H\left[(H_T - H_w) + (r_H - 1)\frac{V_e^2}{2}\right] \tag{4}$$

introducing the relation $H_T = H_s + \frac{V_e^2}{2}$ which defines the total enthalpy H_T. This results in a form which will be very useful when dissociation occurring in the boundary layer has to be taken into account.

In recent aeronautical developments, velocities have become very high so that, correspondingly, temperatures encountered within the boundary layer surrounding a flying vehicle or between the shock and the boundary layer are of a magnitude which causes at first the oxygen and later on the nitrogen in the air to dissociate into atoms. Similar conditions are encountered in the exhaust gases of rockets. An analysis for laminar Couette flow of a gas with constant properties (or with density times viscosity equal a constant and Pr = const) and with dissociation of molecules into atoms led to the result that the heat flow into the stationary wall can be described by the following equation

$$q = \frac{k}{c_p b}\left[(H_{T1} - H_w) + (Pr - 1)\frac{V_1^2}{2} + (Le - 1)H_D(W_{a1} - W_{aw})\right] \tag{5}$$

provided that thermodynamic dissociation equilibrium is established everywhere in the flow field [14]. In this equation, b is the distance of the two walls, the index one indicates values existing at the wall moving with a velocity V_1, index w indicates conditions at the stationary wall, Le denotes the Lewis number (the ratio of mass diffusion coefficient of the atoms to the thermal diffusivity), H_D denotes the dissociation enthalpy per unit mass of atoms, and W_a the mass fraction of atoms. The group $k/c_p b$ is the heat transfer coefficient defined by (1) for laminar Couette flow of a one-component fluid without dissociation. The equation (5) exhibits a remarkable parallelity as to the effect of kinetic energy and chemical energy on heat transfer. The term in square brackets can be considered as the driving potential, the first term within round brackets describes the potential for a fluid with Pr and Le equal one, the second term gives a correction for the influence of Prandtl number on the kinetic potential, and the third term gives the correction describing the influence of Lewis number on the chemical potential. The second term becomes zero for a Prandtl number equal to one and the third term for a Lewis number equal to one so that, for such a fluid, heat transfer is the same regardless whether dissociation occurs or not. Prandtl and Lewis numbers in gases have usually values not far from one.

The equation (5) can be generalized so that it becomes also applicable to boundary layer flow

$$q = h_H\left[(H_T - H_w) + (r_{HV} - 1)\frac{V_e^2}{2} + (r_{HD} - 1)H_D(W_{ae} - W_{aw})\right] \tag{6}$$

The index e indicates now conditions at the outer edge of the boundary layer (corresponding to the values 1 at the moving wall for Couette flow). r_{HV} is the enthalpy recovery factor for kinetic energy and r_{HD} the recovery factor for chemical energy. A comparison with (5) shows that for Couette flow $r_{HV} = Pr$ and $r_{HD} = Le$. For laminar boundary layer flow without dissociation $r_{HV} = \sqrt{Pr}$, and for a turbulent boundary layer $r_{HV} = \sqrt[3]{Pr}$. Analogy suggests that $r_{HD} = \sqrt{Le}$ for a laminar boundary layer and $r_{HD} = \sqrt[3]{Le}$ for a turbulent one. This is supported by the fact that a boundary layer analysis in [15] for rotationally symmetric laminar flow of air near a stagnation point resulted in the value $r_{HD} = Le^{0.52}$, again under the assumption of local dissociation equilibrium.

The heat transfer coefficient h_H for laminar and turbulent flow over a flat plate can be obtained from the relations

$$\frac{St_H}{C_f/2} = Pr^{-2/3}, \quad St_H = \frac{h_H}{\rho V_e} \tag{7}$$

Introduction of the properties at a reference enthalpy as given by (2) should again take into account property variations of the dissociating gas [16]. One point has to be kept in mind in such a calculation with regard to the property values necessary for the evaluation of (6) and (7). There exists a certain arbitrariness in the definition of such properties in a dissociating gas as indicated in Fig. 9. This figure is an example of equilibrium properties for air contained in [17]. The specific heat c_p can be calculated either including the heat of dissociation or excluding it. The first value is indicated by c'_p and the second value by c_p. c_{po} is the specific heat as it is calculated from simple kinetic theory. One notices that the differences in the specific heat values c_p and c'_p are very large. In a similar manner, two different values can be defined for the heat conductivity. The value k' includes enthalpy transport by diffusion; whereas the value k contains only the energy transport by thermal conduction. k_o is again the heat conductivity as calculated by simple kinetic theory. The first hump in the values c'_p and k' occurs in the temperature range in which oxygen dissociates, the second hump is caused by nitrogen dissociation. The dashed values apply to a change of state in which thermodynamic equilibrium is maintained, whereas the values without dash hold for a change of state with frozen chemical composition. The values c_p and k and a Prandtl number based on these values have to be used in an evaluation of (7). Thermodynamic equilibrium enthalpies (including dissociation), however, have to be introduced for the values H_T and H_w.

In some situations, fluid particles move so fast through the boundary layer that time may be too short at any location to establish local equilibrium. The heat transfer depends, then, on the behavior of the solid surface. Local equilibrium establishes itself at the surface when it is strongly catalytic. Equations (6) and (7) describe heat transfer for this condition when the thermodynamic equilibrium enthalpies are introduced for H_w and when the recovery factor r_{HD} is slightly changed. Reference [15], for instance, reports a value $r_{HD} = (Le)^{0.65}$ for hypersonic flow of air near a stagnation point, assuming thermodynamic equilibrium outside the boundary layer, frozen condition within the boundary layer and a catalytic surface. For a Lewis number $Le = 1$, equation (6) holds without any change. Heat transfer to a moderately catalytic or non-catalytic wall depends on the degree to which equilibrium is approached in the gas itself as well as on the surface. For the extreme situation that no adjustment of the molecule-atom composition occurs within the boundary layer or at the wall (frozen state), $W_{ae} = W_{aw}$ and (6) describes the heat flux into the surface when enthalpies for frozen state are introduced for H_T and H_w. Heat transfer may be reduced considerably for such a situation as compared to equilibrium.

Heat transfer discussed up to now dealt with steady flow and heat flux conditions. The corresponding relations are, in engineering calculations, generally also used for situations in which velocities or temperatures change with time. This procedure, called quasi-steady, is justified as long as the variation in either velocity or temperature is not extremely rapid. An experimental investigation [18] which has recently become available indicates that, even for a situation for which the quasi-steady assumption does not hold, an estimate on the heat transfer can be made quite readily. The investigation considered heat transfer by free convection on a vertical plate when the plate is originally at the same temperature as the surrounding fluid and when, beginning at a certain moment and afterwards, a locally and timewise constant heat flux from the plate

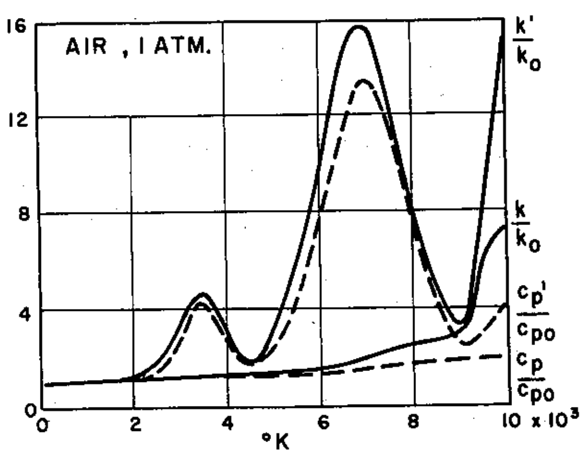

FIG. 9 – VARIATION OF HEAT CONDUCTIVITY (k') AND SPECIFIC HEAT (c'_p) FOR EQUILIBRIUM CONDITION AS WELL AS HEAT CONDUCTIVITY (k) AND SPECIFIC HEAT (c_p) FOR FROZEN COMPOSITION WITH TEMPERATURE.

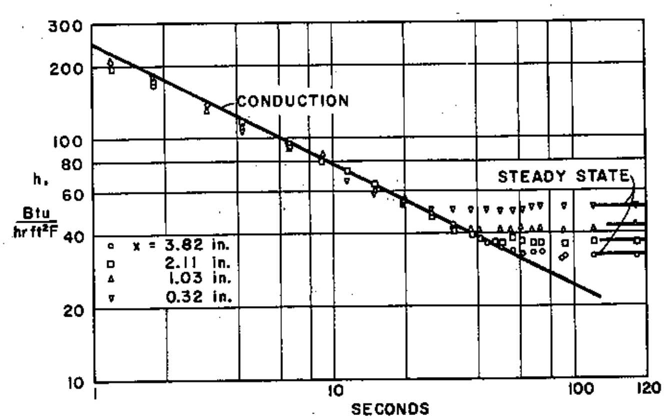

FIG. 10 – HEAT TRANSFER COEFFICIENTS MEASURED FOR UNSTEADY FREE CONVECTION ON A VERTICAL PLATE IN WATER WITH A STEP-WISE CHANGE OF SURFACE HEAT FLUX.

surface into the fluid is produced. Heat transfer coefficients h measured for this situation are presented in Fig. 10 as a function of the time measured from the step increase in the heat flux. The experiments were performed in water in order to have unsteady conditions extended over a time range which could be conveniently observed. The horizontal lines on the right hand side of the figure are heat transfer coefficients calculated for steady state and the inclined line describes the heat flux into the fluid by unsteady conduction only. It can be recognized that the heat transfer in the whole time interval is very well accounted for when the steady state lines are extended to the point of intersection with the conduction line and when the larger one of the two values described by the conduction and the steady state solutions is used. Analyses [19, 20] indicate that a similar situation exists for heat transfer in forced convection with a sudden temperature change.

It has been found that many of the engineering concepts and methods described up to now can be extended to a cooling process termed mass transfer cooling. Such a process is characterized by the fact that a mass flow away from the surface bounding the fluid is generated. This may be done by ejection of the coolant gas through a porous surface (transpiration cooling), or by mass release from the surface through evaporation, sublimation, or some chemical reaction (film cooling, ablation cooling). Time does not permit to discuss the calculation procedures for this cooling method [21]. It has also been found that chemical reactions occurring within the boundary layer are accounted for in (6) when the enthalpies include the heats of reaction [22].

The use of a constant property fluid has also led to a model which allows a prediction of heat transfer for the film cooling process. In such a process, a cooling fluid is injected through one or several slots into the main stream. The sketch in Fig. 11 indicates this. The injection will have the effect to distort the boundary layer and also to cool it and the plate surface in the downstream region, provided the temperature T_s of the coolant is lower than the temperature T_e in the main stream. It has been found that the distortion of the velocity field in the boundary layer extends a relatively short distance downstream. Beyond this region, the cooling process should be similar to one obtained when the coolant ejection through the slot is replaced by a line heat sink at the proper location. The momentum equation of the boundary layer for a constant property fluid stays then unchanged by the presence of the heat sink and the following procedure for the calculation of such a film cooling process can be derived from the fact that the energy equation is linear [23]. The adiabatic temperature $T_{w,ad}$, which the surface assumes under the influence of the sink for the condition that the heat flux through the wall surface itself is zero, is used to define a cooling effectiveness

$$\eta = \frac{T_e - T_{w,ad}}{T_e - T_s} \tag{8}$$

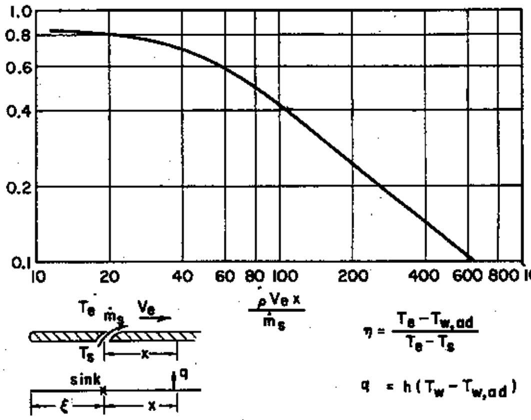

FIG. 11 – EFFECTIVENESS η OF AIR TO AIR FILM COOLING AS FUNCTION OF THE INJECTION PARAMETER.

This cooling effectiveness is described by the following relation [6]

$$\eta = K(Pr)^{2/3}\left(\frac{\dot{m}_s}{\mu}\right)^{0.2}\left(\frac{\rho V_e\, x}{\dot{m}_s}\right)^{-0.8}\left[1-\left(\frac{\xi}{x+\xi}\right)^{9/10}\right]^{-8/10} \tag{9}$$

In this equation, $\dot{m}_s$ is the mass flow of coolant through the slot per unit length and time, V_e the velocity outside the boundary layer, μ viscosity, and ρ density. The coordinates x and ξ are indicated in Fig. 11. The constant K is best determined from experiments. The term $\dot{m}_s/\mu$ can be interpreted as a slot Reynolds number. For the situation that heat is additionally removed from the plate surface through the wall, the magnitude of this heat flux q per unit area and time is given by the equation

$$q = h\,(T_w - T_{w,ad}) \tag{10}$$

in which T_w indicates the actual wall surface temperature. The heat transfer coefficient h has the same value as it would exist at the location under consideration if the heat sink were not present. The effect of the heat sink appears, therefore, in this equation only in the way that the fluid temperature T_e is replaced by the adiabatic wall temperature $T_{w,ad}$. Measured effectiveness values for the film cooling process are presented in Fig. 11. The experiments were made with air of low subsonic velocities flowing with a turbulent boundary layer over a flat plate [24]. The equation (9) describes these measured coefficients satisfactorily for the values of the parameter on the abscissa for which the effectiveness curve has become linear. The validity of (10) was also verified by experiments. The fact that the described calculation procedure gives results in good agreement with measured values indicates again the usefulness of a model utilizing a constant property fluid for convective heat transfer analyses.

It is hoped that this survey, which had to be restricted to major effects, nevertheless will be useful in demonstrating the main ideas which contributed to a development of our knowledge of convective heat transfer processes during the last decade. Some aspects like heat transfer in low Prandtl number fluids (liquid metals) have not been included, because the basic understanding had already been developed prior to the period under consideration; others, like heat transfer near the critical state or in separated flow, have been omitted because the concepts which form the basis of their analysis are still in the early stages of their development.

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