# Joseph Fourier: The Man and His Achievements

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> S.T. Ro Seoul National University Seoul, Korea stro@snu.ac.kr

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- Who Is Fourier
- Political and Social Environment
- What Fourier Did
- How Fourier Influenced

# Jean Baptiste Joseph Fourier



Born: 21 March 1768 in Auxerre, Bourgogne, France Died: 16 May 1830 in Paris, France at the age of 62



# **Joseph Fourier**

- 9th of the 12 children
- Parents died when he was 9 and 10 years old
- 1780(12) Ecole Royale Militaire of Auxerre Lycee Fourier in 1968
- 1782(14) By the age of 14, completed a study of the six volumes of Bézout's *Cours de mathematique* 
  - Bézout (1730-1783): Mathematician, Educator
  - Text translated into English and used for Harvard calculus

• 1783(15) Received the first prize for his study of Bossut's Méchanique en général 
 Fourier
 1768 – 1830

 Napoleon
 1769 – 1821

• 1787(19) – 1789(21) Benedictine abbey of St. Benoit-sur-Loire

Fourier's letter :

"Yesterday was my 21st birthday, at that age Newton and Pascal had already acquired many claims to immortality."

1789(21) French Revolution

• 1790(22) Teacher at the Benedictine College, Ecole Royale Militaire of Auxerre

### 1769 Napoleone Buonaparte was born.

- 1778(9) At age nine, Napoleon is sent to Collège militaire royal de Brienne in Paris. While there, he distinguishes himself by his taste for mathematics and geography.
- 1784(5) Napoleon enters l'Ecole militaire royale de Paris in Paris.
- 1785(16) Napoleon becomes second lieutenant.
- 1789(20) French Revolution
- 1792(23) Napoleon promoted to Captain
- 1795(26) Napoleon is named général de division.
- 1796(27) Napoleon is named General in Charge of the Army of Italy.
- 1798(29) Napoleon heads a French expeditionary force into Egypt.
- 1799(30) French soldiers discover the Rosetta Stone.
- 1799(31) Napoleon becomes First Consul (Premier Consul).
- 1802(33) Napoleon named Consul for life
- 1814(45) Napoleon abdicates and is exiled to Elba.
- 1815(46) March to Paris. The "100 Days". Deported to Santa Helena.
  - Napoleon Bonaparte dies.

- 1793(25) Involved in politics. Joined the local Revolutionary Committee Attempted to resign from the committee, but failed
- 1794(26) Arrested and imprisoned, and released
- 1794(26) Nominated to study at the Ecole Normale in Paris (teachers' institute)
- 1795(27) Studied at the Ecole Normale and taught by Lagrange and Laplace
  - Taught at the College de France.
  - Excellent relation with Lagrange, Laplace and Monge. Appointed at the Ecole Centrale des Travaux Publiques (later Ecole Polytechnique) under the direction of Lazare Carnot
  - Arrested, imprisoned and freed

### • 1795(27) Back to teach at the Ecole Polytechnique (Sept. 1st)

 1798(30) Joined Napoleon's army to Egypt as Scientific Advisor with Monge and Malus

In Egypt, Fourier

- Acted as an administrator in French type political institutions,
- Established educational facilities,
- Carried out archeological explorations,
- Found the Cairo Institute and became the Secretary to the Institute.

 1801(33) Returned to France Resumed Professor of Analysis at the Ecole

1802(34) Asked by Napoleon to serve as the Prefect of the Department of Isere Grenoble.

### Fourier's work in Egypt

- A memoir upon the general solution of algebraic equation
- Researches on the methods of elimination
- The demonstration of a new theorem of algebra
- A memoir upon indeterminate analysis
- Studies in general mechanics
- A technical and historical work upon the aqueduct which conveys the waters of the Nile to Cairo
- Reflections upon the oases
- A plan of statistical researches to be undertaken with respect to the State of Egypt
- An intended exploration of the site of the ancient Memphis and of the whole extent of burial places
- A descriptive account of the revolutions and manners of Egypt from very early times
- A description of a machine designed to promote irrigation and which was to be driven by the power of wind.

### His works in Grenoble also include :

- The operation to drain the swamps of Bourgoin,
- The construction of a highway from Grenoble to Turin
- The work on the Description of Egypt

![](_page_11_Picture_0.jpeg)

![](_page_12_Figure_0.jpeg)

### Work on the theory of heat "On the Propagation of Heat in Solid Bodies"

- 234 pages of book, the Institut de France in Paris -

- Read to the Paris Institute on December 21st, 1807
- 1804(36) 1807(39) in Grenoble and probably during in Egypt
- Review Committee : Lagrange, Laplace, Monge and Lacroix
- Results: Highly recognized mathematical analysis of physical phenomena outside the terms of reference of Newton's law of gravitation, <u>but</u>
  - Lagrange and Laplace in 1808 *Fourier's expansion of functions as trigonometric series*Biot

*Derivation of the equations of transfer of heat* Reference to Biot's 1804 paper

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# Fourier's Original Writing

### • January 2<sup>nd</sup>, 1810

The Paris Institute set the 1811 mathematics prize on the subject of the propagation of heat in solid bodies to be in by 1811 October 1<sup>st</sup> :

"Give the mathematical theory of heat and compare the result of this theory with exact experiments."

Fourier submitted the 1807 memoir with additional work on the cooling of infinite solids and terrestrial and radiant heat

Award(Examiners) Committee:

- Lagrange, Laplace, Malus, Haüy and Legendre

"This work contains the true differential equations of the transmission of heat, both in the interior of the bodies and at their surface and the novelty of the crown this work, observing, however, that the manner of arriving at its equations is not free from difficulties and its analysis of integration still leaves something to be desired, both relative to its generality and on the side of rigour."

"...The author of this paper is the Baron Fourier, Member of Legion of Honour, Baron of the Empire."

The prize was **awarded to Fourier**, but with criticism : Good work to be crowned to fit the class of the Institute, but something further needed on the score of generality and rigor. No publication in the journals of the Institute.

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  - Napoleon Bonaparte dies.

![](_page_18_Figure_0.jpeg)

![](_page_19_Picture_0.jpeg)

Napoleon's triumph return from Elba

 1815(47) Appointed as Prefect of the Rhone centered at Lyon, but resigned soon before the end of Napoleon's Hundred Days and moved to Paris to follow his intellectual life and get his prize paper and book printed

### • 1817(49) Elected to the Académie des Sciences

• 1822(54) Secretaire perpetual to the Académie des Sciences to succeed Delambre.

Published Fourier's prize winning essay "Theorie analytique de la chaleur"

Delambre arranged the publication of Fourier's work before his death and Fourier's prize winning essay was published.

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 $\frac{\pi}{2} F(z) = \int_{z}^{z} dz \cos \varphi z \int_{z}^{z} dz F(a) \cos \varphi a.$ 

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 $a\,\sin x+b\,\sin 2x+c\,\sin 3x+d\,\sin 4x+dc.,$ which is first supposed to contain only odd powers of x218. Different expression of the same development. Application to the function  $e^{a}-e^{-a}$  219—221. Any function whitever  $\phi\left(a\right)$  may be developed under the form  $a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + ... + a_1 \sin 4x + dc.$ 

The value of the general coefficient  $\alpha_i$  is  $\frac{2}{\pi} \int_x^{\pi} dx \phi(x) \sin ix$ . Whence we ferive the very simple theorem  $\frac{\pi}{2}\phi(t) = \sin\pi\int_{0}^{t}ds\,\phi(s)\sin s + \sin2\pi\int_{0}^{t}ds\,\phi(s)\sin2s + \sin2s\int_{0}^{t}ds\,\phi(s)\sin3s + ise_{s}$ 

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 $\int_0^\infty \frac{dq}{q} \sin q \cos q x \text{ is } \frac{1}{2} x,$ if we give to x a value included between 1 and -1. The definite integral has a nul value, if x is not included between The definite integers I and -1. Application to the case in which the heating given results from the final state which heating of a source of heat determines Discontinuous values of the function expressed by the integral

\$51-469. We consider the lines measured of heat in a line whose initial temperatures are represented by  $\tau \rightarrow f(v)$  at the distance ar to the right of the origin, and by  $\tau \rightarrow f(v)$  is the distance or to be first of the origin. Representation of the variable temperature at any point. The robust interval tem the analysis which argument the morement of heat in an derived from the analysis which expresses the more ment of laws in an infinite line . 6. 6. 554. Expression of the variable temperatures when the initial state of the part basels is required by a set of the variable statement of the state of the state

 $\frac{\pi}{2}f(x) = \int_{0}^{\infty} dq \sin qx \int_{0}^{\infty} da f(a) \sin qx.$ 

The function f(x) satisfies the condition f(-z) = -f(z)Min-Min. Use of the preceding results. Front of the theorem expressed

 $\pi\phi\left(a\right) = \int_{-\infty}^{+\infty} da \, \phi\left(a\right) \int_{-\infty}^{\infty} dq \cos\left(qx - qa\right).$ 

This equation is evidently included in equation (II) stated in Ast. 234, (See Act. 207)

(66, 447, 107).
 (7).
 (8). The foregoing system of the variable movement of basis in an infinite line, we point of which is arbitration of the secondard temperature.
 (8). The same points of which is an arbitration of by mass of a souther form of the integral. Forenation of the integral of the second souther form of the integral. Forenation of the solution to an infinite prim, whose initial temperatures are unit. Examplishes the synthese of the difference of heat.
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7-369. The same integral applies to the problem of the diffusion of heat. The solution which we derive from it agrees with that which has been stated in Articles 347, 348

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are: On Them as hand body is planned as an arriform succession at a lower ture by respected in the other planness spectra of the structure of the temperature of the nervice scenarios as preperioant to the ensure of the improvements of the nervice scenarios preperioant to the ensure of the improvements of the nervice scenarios preperiod the molecular of the scenarios of the structure of the scenarios of the scenarios of the scenarios of the scenarios where the scenarios of the scenarios on variations as the conditional by comparing the neural networks for variations at the conditional by comparing the neural networks of the scenarios with very exact experiments

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OF THE UNIFORM AND LINEAR MOVEMENT OF HEAT. or two screeks are before a setting of 100.5.
C. To permanent inspection will instead between two possible phases maintained at first inspectives, see represent by the other setting phase instances for the distance of the setting of 0.6. (7). Obting and attempts the set of 0.6. (8).
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(6). The other setting of 0.6. (8).
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The value of P is  $\frac{\sigma}{s}\left(\frac{g}{k}+\frac{g c}{R}+\frac{g}{R}\right)$  , as in the temperature of the internal Let use or  $u = \chi_{-}^{-} (1 + \chi_{-}^{-}))^{-1}$  is its proposed on the matrix of the form of the state of the states of the st

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ANT. 224, 225. Second theorem on the development of functions in trigono-metrical series:  $\frac{\pi}{2} \psi(s) = \Sigma_{s=0}^{b=\infty} \cos i s \int_0^{\pi} ds \cos i s \psi(s).$ 

 $\frac{1}{4} \pi \sin \pi = \frac{1}{2} - \frac{\cos 2\pi}{1.3} - \frac{\cos 4\pi}{3.5} - \frac{\cos 4\pi}{5.7} - 40.$  (190) 1-230. The precoding theorems are applicable to discontinuous functions, and solve the problems which are based upon the analysis of Daniel Bernoulli in the problem of vibrating code. The value of the series,

sin x versis  $a + \frac{1}{2} \sin 2a$  versis  $2a + \frac{1}{2} \sin 3x$  versis 3a + da. is  $\frac{\pi}{n}$ , if we attribute to n a quantity greater than 0 and less than n; and

value of the series is 0, if x is any quantity included between a and  $\frac{1}{2}w$ . Solution to other remarkable enaugher; curvel lines or surfaces which which in a part of their course, and differ in all the other parts \_033. Any function whatever, F(s), may be developed in the form

 $F(x) = A + \left\{ \begin{array}{l} a_1 \cos x + a_2 \cos 2x + a_2 \cos 3x + \delta x, \\ b_1 \sin x + b_2 \sin 2x + b_2 \sin 2x + \delta x \\ \end{array} \right.$ Each of the coefficients is a definite integral. We have in general

 $2\pi A = \int_{-\pi}^{+\pi} dx F(s),$   $\pi a_i = \int_{-\pi}^{+\pi} dx F(s) \cos is,$  $eb_1 = \int_{-\pi}^{+\pi} dx P(x) \sin ix.$ 

We thus form the general theorem, which is one of the shief elem

 $2\pi F(s) = \Sigma^{s-s-s} \left(\cos i s \int_{-\infty}^{+\infty} ds F(s) \cos i s + \sin i s \int_{-\infty}^{+\infty} ds F(s) \sin i s \right),$ 

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Tau Housey Teasurantian on as because Sources (a) 100 million of the prime distributes letel through-ent flow which mass. The temperature at a distant point rises pr-genaicity, arrives at its greatest value, and then decrement. The time

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d successive envelopes. Remarkable effects of the separation of the es. These results applicable to many different problems SECTION VIL OF THE UNIVORM MOVEMENT OF HEAT IN THESE DIS.

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of the ratio  $\frac{h}{k}$ : we have  $\frac{v_1 + v_2}{v_1} = q$ ,  $w^2 - qw + 1 = 0$ , and  $\frac{h}{k} = \frac{S}{I} \left( \frac{\log w}{h \log w} \right)^2$ 

The distance between two consecutive points is  $\lambda_i$  and log  $\omega$  is the decimal logarithm of one of the two values of  $\omega$ 

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 $\frac{d^{3}v}{dr^{2}} + \frac{d^{3}v}{dr^{3}} + \frac{d^{3}v}{dr^{3}} = 0$ ;

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This remark leads to the equation  $v = A \frac{\sin n\pi}{2} e^{-K_0 \theta}$ , which expresses

the simple movement of heat in the sphere. The number a has an

infinity of values given by the definite equation  $\frac{nX}{4m\pi X} = 1 - hX$ . The

ratios of the sphere is denoted by X, and the radiance any concention phere, where trapportees is a sub-the large of the thirt of the A sub-trapport of the sphere of the thermal state of the sphere of the size of the subject to disclose the nature of the definite equation, the limits and values of air rots -292. Formation of the general solution jam is and the solid  $\lambda$ . Application to the cases is which the sphere has been basid by a per-logical immerica radius of the sphere is denoted by X, and the r sphere, where temperature is v after the lapse

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 $\frac{dv}{dt}=\frac{d^4v}{dt^3}.$ 

 $f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} da f(x) \int_{-\infty}^{++\infty} dp \cos(px - pa) \dots (B) \dots (b)$ 

417. Any limits a and b may be taken for the integral with respect to a. These limits are those of the values of x which correspond to existing values of the function /|x|. Every other value of x gives a nul result

 $f(z) = \frac{1}{2\pi} \sum_{i=-\infty}^{i_{m+m}} \int_{-\infty}^{+\infty} da f(a) \cos \frac{2i\pi}{X} (z-a),$ 

 $\frac{d^2 \cdot f(x)}{dx^4}$ 

23. Construction which serves to prove the general equation. Consequences relative to the extent of equations of this kind, to the values of f(a) which entropy of a the limits of a, to the infinities where of f(a) 1-477. The method which exceed is is determining by definite integration is defined in the distribution of a function of a sudar tie form.

 $e\phi(\mu, x) + b\phi(\mu, x) + e\phi(\mu, x) + de$ 

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 $\sigma \ f(c)$  . The same remark applies to the general equation

421. Application to the equation  $\frac{d^3q}{dx^3} + \frac{d^3q}{dy^3} = 0$ 

422. General expression of the flu

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 $\frac{d\sigma}{dt} = \frac{K}{CD} \left( \frac{d^3 \sigma}{dx^3} + \frac{2}{x} \frac{d\sigma}{dx} \right) \quad . \qquad .$ 

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-162. Purplamental considerations on the nature of the quantilities s, t, v, K, b, C, D, which enter into all the analytical expressions of the Theory of Hest. Rach of these quantities has an exponent of dimension which relates to the length, or to the duration, or to the temperature. These exponents see focus by making the units of measure very.

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163.—168. The constant temperatures of a rectangular plate included be-tween two parallel infinito sides, maintained at the temperature 0, are

107—170. If we consider the state of the plate at a very great distance from the transverse edge, the ratio of the temperatures of two points whose coordinates area v<sub>i</sub>, y and v<sub>i</sub>, y charges according as the value of y increases; i v<sub>i</sub> and v<sub>i</sub> preserving that respective values. The ratio has

increases  $p_1$  and  $p_2$  preserving that respective values. The ratio has a limit to which it approaches more and more, and when y is infinite, it is expressed by the product of a function of x and of a function of y. This remark suffices to disclose the eventral form of x, namely.

 $v = \Sigma_{(-1)}^{\ell=0} a_{\ell} e^{-(2\ell-1)\cdot t}$ , cos (2l - 1).y.

It is easy to ascertain how the movement of heat in the plate is

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 $u = \frac{1}{\pi} \int_0^{\pi} dr \cos \left( z \sqrt{g} \sin r \right);$ 

 $a + bc + c \frac{a^3}{2} + d \frac{x^2}{2.3} + do.$ 

 $\frac{1}{\pi}\int_{a}^{a} du \phi(t \sin u).$ 

 $a + \frac{ct^0}{2^2} + \frac{ct^4}{2^2, 4^2} + \frac{\rho t^4}{2^2, 4^2, 6^2} + 4cc_{\alpha}$ 

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Assumpts. An attick "On the linear motion of hast, Part IL", writes by fit We Treesees under the signature N.N. will be found in the Constroker Matchanization Journel, Val. ILL, pp. 90–911, and in Vol. L of the Anthree Matchanization Journel, Val. ILL, pp. 90–911, and in Vol. 1 of the Anthree Trainston of hast is an infinite still, be concluded by a phase, may be suppared to have result, by conclusion, (k,F)

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and the definite equation is  $h_0 + \frac{du}{dx} = 0$ , giving to x its complete value X. 294

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 $\{-1\}^{n-2}_{\geq 3}$  when i is of form 2n+3

ite value,  $\frac{b}{-}(1-e^{-ye})$ 

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expressed by the equation  $\frac{d^4y}{dx^4} + \frac{d^4y}{dx^4} = 0$ .

ANP. 156, 156. In applying the general equation (A) to the case of the op and of the sphere, we find the same equations as these of Section and of Section II. of this chapter

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92, 83. The permanent temperatures of a solid enclosed between aix rec-tangular planes use expressed by the equation. v = A + ax + by + cz.

n, y, z are the coordinates of any point, whose temperature is  $v_1 \cdot A_1$ , a,  $b_2 \in are constant numbers. If the extreme planes are maintained by any constant statistical temperatures which existing the preceding equation, the final system of all the internal temperatures will be expressed by the man estation:$ ne optation . Measure of the flow of heat in this prism

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16−09. The variable system of temperatures of a solid is repport to be expressed by the equation v − F (n, y, n, t), where n denotes the resulties temperature with words or the system of the the time that the object is the point whose section are n, y, n. Formation of the form of the data of the solid line of the solid line to solid line (solid line) and the solid line (solid line) and the solid line) and the solid line (solid line) and the solid line) are solid line (solid line). The solid line (solid line) are solid line) and the solid line (solid line) are solid line) and the solid line (solid line) and the solid line) are solid line (solid line). The solid line (solid line) are solid line) and the solid line (solid line) are solid line).

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SECTION L Equation of the Values Movement of Heat is a Riss.

-105. The variable movement of heat in a ring is expressed by the  $\frac{dv}{dt} = \frac{K}{CD} \frac{d^3v}{dx^3} - \frac{M}{CDS} v,$ The are x measures the distance of a sortion from the origin  $O_{\pm} v$  is the temperature which that acction acquires after the layes of the time  $t_{\pm}$  $N_{\pm}$ , O, D, A are the specific accellisaties z is it to area of the scetcher, by the revolution of which the ring is generated; t is the perimeter of the sceton

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moreover of least is an indicate shift  $\partial B_{ii}$ . Two other three  $i_{ij}$  shall be the backgraph, which are derived, like the prescaling form, from the indegral (a)  $\partial B_{ii}$ . The other here  $\partial B_{ii}$  shows the other independence of the time  $A_i$ . Records derivative and the time  $A_i$ . Records derivative  $\partial B_{ii}$  shows the relative function of I.  $\partial B_{ii}$ . Noticing asymptotics to the representation of these derivatives  $\partial B_{ii}$  shows the shows the relative  $\partial B_{ii}$  shows the function  $\partial B_{ii}$ . Applied the shows the relative  $\partial B_{ii}$  shows the function  $\partial B_{ii}$ . Applied the the spectrum i

 $\frac{d^3\sigma}{dt^2} + \frac{d^4\sigma}{dx^4} + 2\frac{d^4\sigma}{dx^4, dx^4} + \frac{d^4\sigma}{dx^4} = 0.....(c),$ 

404. Use of the theorem E of Article 363, to form the integral of equation (/) of the preceding Article Use of the same theorem to form the integral of equation (d) which

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 $\frac{d^4 \pi}{d\xi^2} = \frac{d^4 \pi}{dx^4} + \frac{d^4 \pi}{dx^5}, \dots, (c), \quad \text{ and } \frac{d^4 \pi}{d\xi^4} + \frac{d^4 \pi}{dx^4} = 0, \dots, (d) \ . \ . \ . \ 404$ 

 $\frac{dv}{dt} = a \frac{d^2 v}{dx^2} + b \frac{d^2 v}{dx^2} + c \frac{d^2 v}{dx^2} + \delta v \dots (f) \quad , \quad , \quad , \quad 405$ 

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#### CHAPTER IV.

#### OF THE LINEAR AND VARIED MOVEMENT OF HEAT IN A RING.

#### SECTION I.

#### General solution of the problem.

238. THE equation which expresses the movement of heat in a ring has been stated in Article 105; it is

$$\frac{dv}{dt} = \frac{K}{CD} \frac{d^2v}{dx^2} - \frac{hl}{CDS}v \dots (b).$$

The problem is now to integrate this equation: we may write it simply

$$\frac{dv}{dt} = k \frac{d^2v}{dx^2} - hv,$$

wherein k represents  $\frac{K}{CD}$ , and h represents  $\frac{hl}{CDS}$ , x denotes the length of the arc included between a point m of the ring and the origin O, and v is the temperature which would be observed at the point m after a given time t. We first assume  $v = e^{-ht}u$ , u being a new unknown, whence we deduce  $\frac{du}{dt} = k \frac{d^2u}{dx^2}$ ; now this equation belongs to the case in which the radiation is nul at the surface, since it may be derived from the preceding equation by making h = 0: we conclude from it that the different points of the ring are cooled successively, by the action of the medium, without this circumstance disturbing in any manner the law of the distribution of the heat.

In fact on integrating the equation  $\frac{du}{dt} = k \frac{d^3u}{dx^2}$ , we should find the values of u which correspond to different points of the

# Fourier's Ring

# From Carslaw and Jaeger, P.160(1973)

One of the simplest and most suggestive problems in the conduction of heat, when the temperature depends only upon one coordinate and the time, is Fourier's problem of the ring. This problem is also of special interest, as it was the first to which Fourier applied his mathematical theory, and for which the results of his mathematical investigations were compared with the facts of experiment.<sup>†</sup>

The ring consists of a small cross-section twisted into a circle (or other closed curve). Then with the notation and assumptions of § 4.2 the differential equation for the temperature in the ring is 4.2 (4), that is

$$\frac{\partial v}{\partial t} = \kappa \frac{\partial^2 v}{\partial x^2} - \nu v. \tag{1}$$

We suppose the length of the ring to be 2l, so that taking the origin at any convenient point we have to solve (1) in the region  $-l \leq x \leq l$ . Since the ring forms a closed curve we do not have boundary conditions at  $x = \pm l$ , but instead the condition that v is to be periodic with period 2l in x, that is

$$v(x,t) = v(x+2nl,t), \quad n = 1, 2, \dots$$
 (2)

I. Initial temperature f(x). No radiation We assume that f(x) can be expanded in the Fourier series

$$f(x) = \sum_{n=0}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}.$$
 (3)

Then

$$v = \sum_{n=0}^{\infty} a_n e^{-\kappa n^2 \pi^2 t/l^2} \cos \frac{n \pi x}{l} + \sum_{n=1}^{\infty} b_n e^{-\kappa n^2 \pi^2 t/l^2} \sin \frac{n \pi x}{l}$$
(4)

satisfies all the conditions of the problem. This may be verified  $\ddagger$  as in § 3.3. The solution for the case of radiation follows on substituting  $v = ue^{-\nu t}$  in (1). † Fourier, Théorie analytique de la chaleur, Chaps. II and IV.

‡ It may also be verified that in this case v and  $\partial v/\partial x$  are continuous at  $x = \pm l$  for t > 0, as they should be since the ring forms a continuous curve. They need not be continuous there when t = 0.

§ Fourier, loc. cit., §§ 107-10.

# What made Fourier's interest and motivation in heat propagation?

- Grenoble and Egypt (?)
- In 1736, Academy of Science of Paris had proposed "the Study of the nature and the Propagation of Fire" as the subject of a prize essay. Euler was crowned with two others.
- Napoleon favored the mathematical sciences and created prizes for physical discoveries.
- Earlier Work
- The French Revolution

• Lagrange's Memoir on "The Nature and Propagation of Sound" (1759)

$$y = 2\int_{0}^{1} \sum_{1}^{\infty} (\sin n\pi x' \sin n\pi x \cos n\pi at) f(x') dx'$$
$$+ \frac{2}{a\pi} \int_{0}^{1} \sum_{1}^{\infty} \frac{1}{n} (\sin n\pi x' \sin n\pi x \sin n\pi at) F(x') dx'$$

where, f(x) : initial displacement, F(x) : initial velocity

at t=0 
$$f(x) = 2\sum_{1}^{\infty} A_n \sin n\pi x$$
  
 $A_n = \int_{0}^{1} \sin n\pi x' f(x') dx'$ 

 Why Lagrange missed? The object of Lagrange was to obtain the functional solution, not the coefficients!

- The French Revolution (Described by G. Cuvier (1769-1832))
  - Reconstruction with demolition
  - Practical popularization of science and to establish its educational and technical importance
  - The Memoirs of the Academy: confined to the measured and concise statements of facts or to theories capable of mathematical verification and treatment
  - Defense and Patriotism:
  - L. Carnot and many other mathematicians and scientists
    New methods of manufacturing, natural resources
    Existing academics and colleges: organized a system of public instruction
    Professors and officers
    A great number of students studied the different branches of knowledge
  - . A great number of students studied the different branches of knowledge and the art of teaching under the greatest masters

- In the 19<sup>th</sup> century
  - The revolutionary transformation of the traditional scientific disciplines into the exact sciences : mathematization of sciences, electricity, magnetism, mechanics, light, **heat**
  - Method of approaches to formulate :
    - . Facts and underlying causes
    - . Facts and observations

### • Publications by Fourier, 1820-1829

Light & Wave Motion	2
Heat	5
Mathematics and Mechanics	16
Total	23

Among 295 paper published by 14 scientists including Laplace, Fourier, Arago, Biot, Poisson, Ampere, Dulong, etc. in the period of 1820-1829, 30 papers are related to Heat (10%).

### Fourier

- Theoretical and experimental physicist
- Mathematician
- Theorie Analytique de la Chaleur
  - On December 21st of 1807, 234-page work
  - On 1822, 433 articles in 541 pages

# The Theory of heat

- 1807 "On the Propagation of Heat in Solid Bodies"
- 1822 "Theorie analytique de la chaleur"
- 1824 Sadi Carnot
- 1840 James Prescott Joule (1818-1889)
- 1842 Julius Robert von Mayer (1814-1878)

### Fourier

- Elegant writer
- Master of good style

Almost no grammatical flaws

"To found the theory, it was in the first place necessary to distinguish and define with precision the elementary properties which determine the action of heat. I then perceived that all the phenomena which depend on this action resolve themselves into a very small number of general and simple facts; whereby every physical problem of this kind is brought back to an investigation of mathematical analysis. From these general facts I have concluded that to determine numerically the most varied movements of heat, it is sufficient to submit each substance to three fundamental observations. Different bodies in fact do not possess in the same degree the power to *contain* heat, *to receive or transmit it across their surfaces*, nor to *conduct* it through the interior of their masses. These are three specific qualities which our theory clearly distinguishes and shows how to measure."

Joseph Fourier, 1822

### Heat Propagation

"But whatever may be the range of mechanical theories, they do not apply to the effects of heat. These make up a special order of phenomena, which cannot be explained by the principles of motion and equilibrium. We have for a long time been in possession of ingenious instruments adapted to measure many of these effects; valuable observations have been collected; but in this manner partial results only have become known, and not the mathematical demonstration of the laws which include them all."

- **0. All motion of heat depends on temperature differences**
- 1. Power of bodies to contain heat
- 2. Power of bodies to receive or transmit heat across their surfaces
- 3. Power to conduct heat through the interior of their masses

### Fourier's achievements are

- Outside the scope of rational and celestial mechanics
- Theory of functions and representation as trigonometric series
- Mathematical analysis of physical phenomena
- Novel treatment and application of linear differential equations to nontrivial boundary value problems with separable spatial and temporal variables
- To distinguish between two kinds of physical behavior action at an interior point and action on a surface boundary
- Equations in a coordinate system appropriate to the problem
  Explicit statements of initial conditions

## Unfavorable receptions

- Rigorous proof for convergence
- Lagrange's and Euler's earlier work
- Scientific rivals
- Isolation from Paris and no regular intellectual contact
- Political and administrative duty

### Fourier's Experimental Work

- Conducted experiments in the period of 1806-1807
- In his 1807 paper,
  - Steady thermal state in annulus
  - Heat diffusion in annulus
  - Heat diffusion in sphere
  - Comparison between sphere and cube on the rate of cooling
  - Error and response of thermometers
- Mercury thermometer: 0°R(Réaumur scale) 80°R
- Heating with Argand lamp
- Time : 3 different clocks 9h21m, 9h21m, 9h 20m
  Room temperature : 19°R or 20°R

the ring (see figure 3), l is the perimeter of the section whose area

Fig. 3,

is S, the coefficient h measures the external conducibility, K the internal conducibility, C the specific capacity for heat, D the density. The line oxx'x'' represents the mean circumference of the armlet, or that line which passes through the centres of figure of all the sections; the distance of a section from the origin o is measured by the

arc whose length is x; R is the radius of the mean circumference.

It is supposed that on account of the small dimensions and of the form of the section, we may consider the temperature at the different points of the same section to be equal. Table I. Experiments on the steady state in annulus.

![](_page_38_Figure_1.jpeg)

 $\theta a, \theta b, \theta c, \theta d, \theta x$  and  $\theta r$ : indications of thermometers a, b, c, d, x and temperature of air on Réaumur's division.

f and f': heat sources. H and K: external and internal conductivity. Th: thermometer.

\* time required from the start of heating to the end of observation.

# After Fourier

- Fourier Series
  - Poisson
  - Cauchy
  - Dirichlet
  - Riemann
- Fourier's law
  - Ohm's law (1826)  $I = \frac{1}{R}V$ - Fick's law (1855)  $\dot{m} = -D\frac{\partial C}{\partial x}$
  - Kelvin

# Conclusion

- Fourier
- Grenoble

# Jean Baptiste Joseph Fourier

![](_page_41_Picture_1.jpeg)