Joseph Fourier:
The Man and His Achievements

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Contents

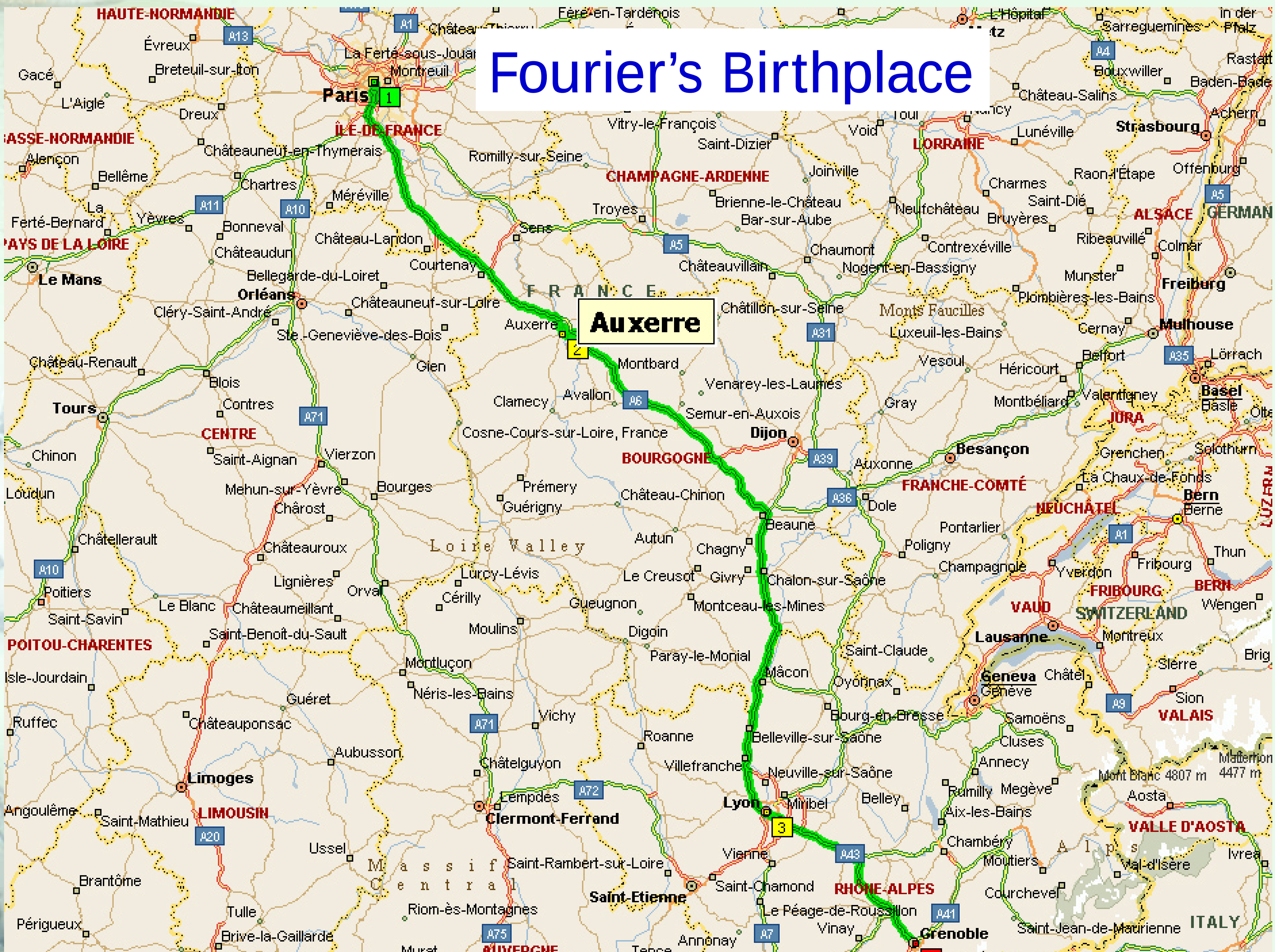
- Who Is Fourier
- Political and Social Environment
- What Fourier Did
- How Fourier Influenced

Jean Baptiste Joseph Fourier

Born: 21 March 1768 in Auxerre, Bourgogne, France

Died: 16 May 1830 in Paris, France at the age of 62

Fourier's Birthplace



Joseph Fourier

- 9th of the 12 children
- Parents died when he was 9 and 10 years old
- 1780(12) Ecole Royale Militaire of Auxerre
  Lycee Fourier in 1968
- 1782(14) By the age of 14, completed a study of the six volumes of Bézout's *Cours de mathematique*
  - Bézout (1730-1783): Mathematician, Educator

  Text translated into English and used for Harvard calculus
- 1783(15) Received the first prize for his study of Bossut's *Méchanique en général*

Fourier	1768 – 1830
Napoleon	1769 – 1821

- 1787(19) – 1789(21) Benedictine abbey of St. Benoit-sur-Loire

  Fourier's letter：

  "Yesterday was my 21st birthday, at that age Newton and Pascal had already acquired many claims to immortality."

- 1789(21) French Revolution
- 1790(22) Teacher at the Benedictine College, Ecole Royale Militaire of Auxerre

1769	Napoleone Buonaparte was born.
1778(9)	At age nine, Napoleon is sent to Collège militaire royal de Brienne in Paris. While there, he distinguishes himself by his taste for mathematics and geography.
1784(5)	Napoleon enters l'Ecole militaire royale de Paris in Paris.
1785(16)	Napoleon becomes second lieutenant.
1789(20)	French Revolution
1792(23)	Napoleon promoted to Captain
1795(26)	Napoleon is named général de division.
1796(27)	Napoleon is named General in Charge of the Army of Italy.
1798(29)	**Napoleon heads a French expeditionary force into Egypt.**
1799(30)	French soldiers discover the Rosetta Stone.
1799(31)	**Napoleon becomes First Consul (Premier Consul).**
1802(33)	Napoleon named Consul for life
1814(45)	Napoleon abdicates and is exiled to Elba.
1815(46)	March to Paris. The “100 Days”. Deported to Santa Helena.
1821	Napoleon Bonaparte dies.

- 1793(25) Involved in politics.
  Joined the local Revolutionary Committee
  Attempted to resign from the committee, but failed
- 1794(26) Arrested and imprisoned, and released
- 1794(26) Nominated to study at the Ecole Normale in Paris (teachers' institute)
- 1795(27) Studied at the Ecole Normale and taught by Lagrange and Laplace
  Taught at the College de France.
  Excellent relation with Lagrange, Laplace and Monge.
  Appointed at the Ecole Centrale des Travaux Publiques (later Ecole Polytechnique) under the direction of Lazare Carnot
  Arrested, imprisoned and freed

- 1795(27) Back to teach at the Ecole Polytechnique (Sept. 1st)
- 1798(30) Joined Napoleon's army to Egypt as Scientific Advisor with Monge and Malus

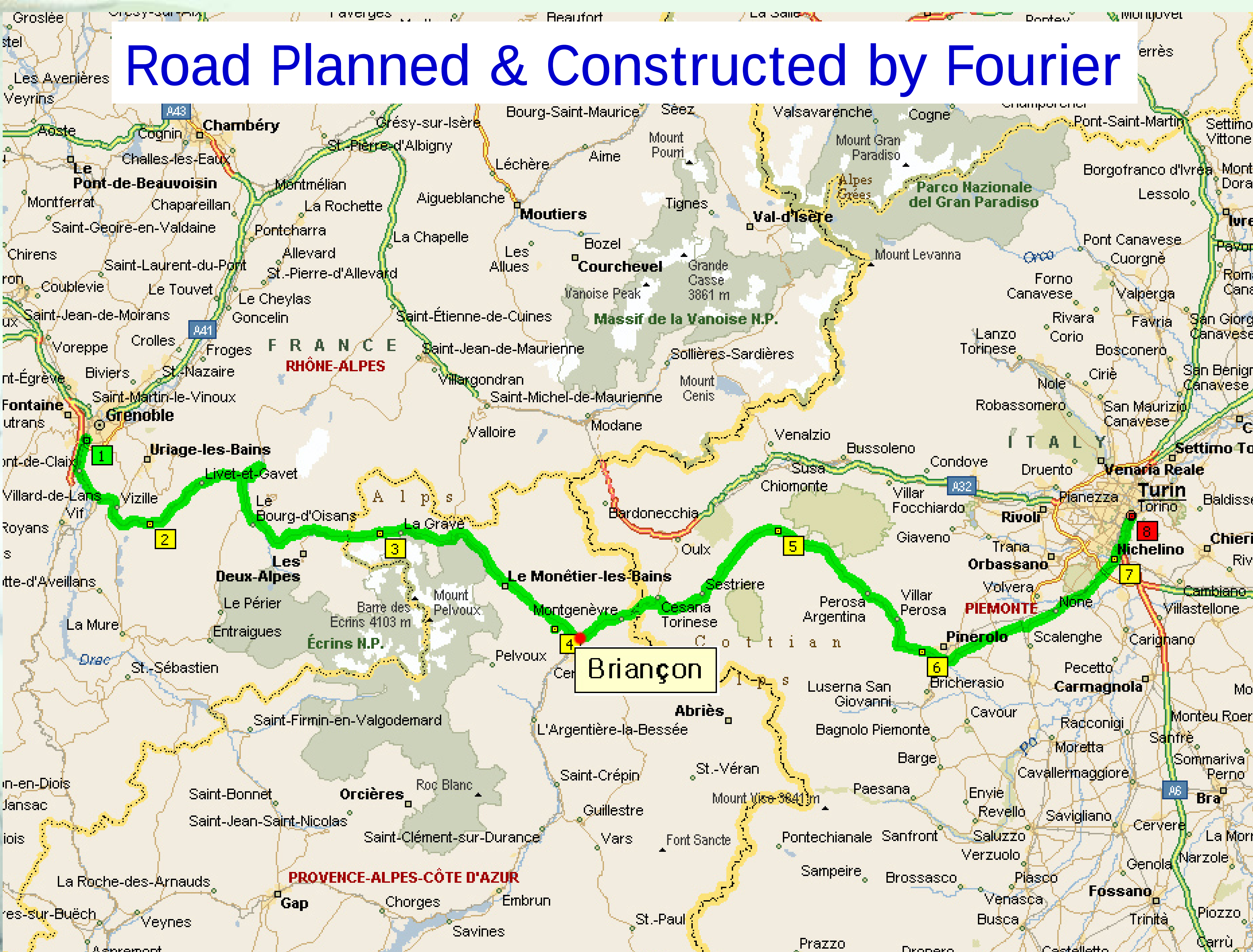
  In Egypt, Fourier
  - Acted as an administrator in French type political institutions,
  - Established educational facilities,
  - Carried out archeological explorations,
  - Found the Cairo Institute and became the Secretary to the Institute.
- 1801(33) Returned to France
  Resumed Professor of Analysis at the Ecole
- 1802(34) Asked by Napoleon to serve as the Prefect of the Department of Isere Grenoble.

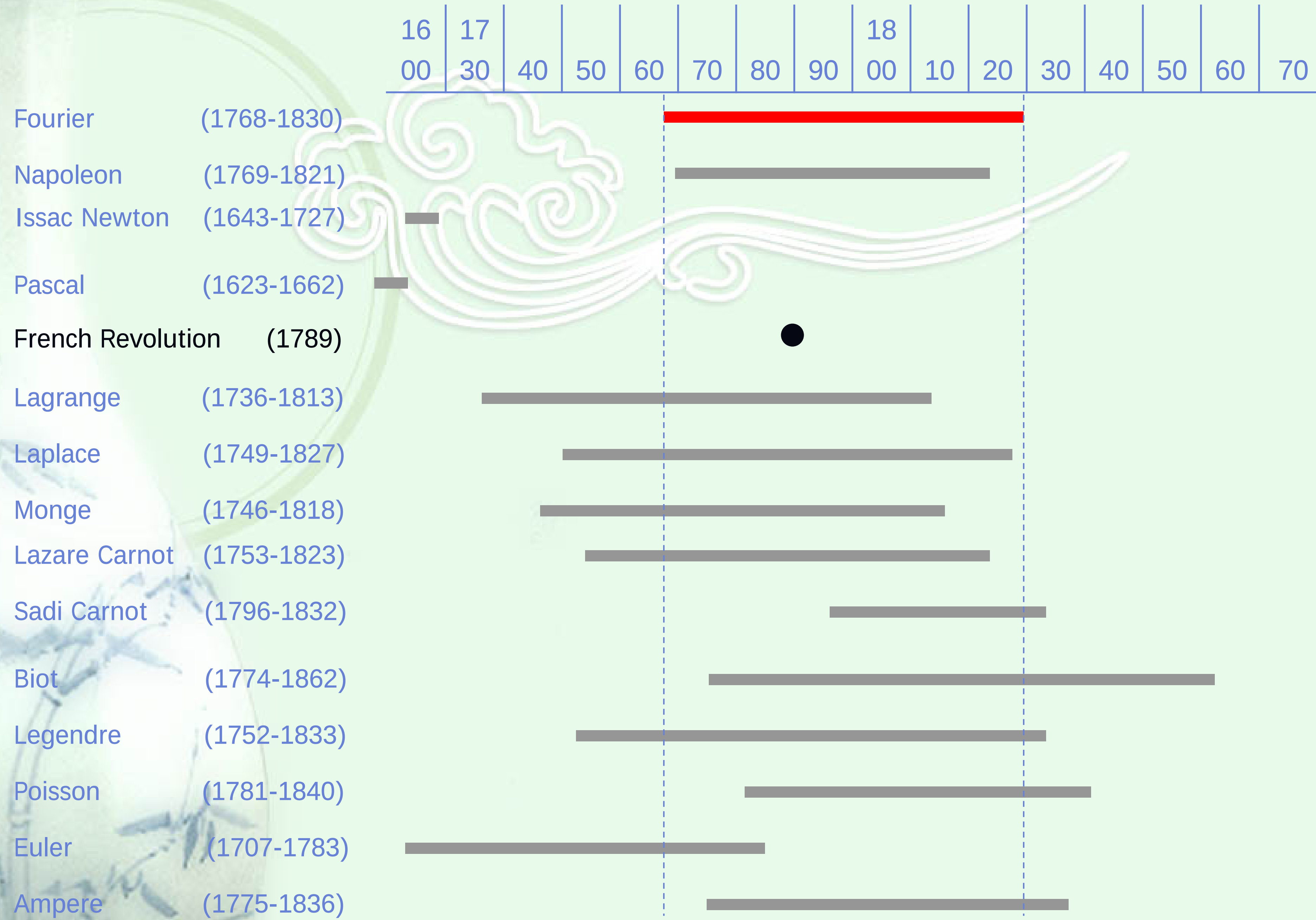
- Fourier’s work in Egypt
  - A memoir upon the general solution of algebraic equation
  - Researches on the methods of elimination
  - The demonstration of a new theorem of algebra
  - A memoir upon indeterminate analysis
  - Studies in general mechanics
  - A technical and historical work upon the aqueduct which conveys the waters of the Nile to Cairo
  - Reflections upon the oases
  - A plan of statistical researches to be undertaken with respect to the State of Egypt
  - An intended exploration of the site of the ancient Memphis and of the whole extent of burial places
  - A descriptive account of the revolutions and manners of Egypt from very early times
  - A description of a machine designed to promote irrigation and which was to be driven by the power of wind.

His works in Grenoble also include :

- The operation to drain the swamps of Bourgoin,
- The construction of a highway from Grenoble to Turin
- The work on the Description of Egypt

Road Planned & Constructed by Fourier

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Fourier (1768-1830)
Napoleon (1769-1821)
Issac Newton (1643-1727)
Pascal (1623-1662)
French Revolution (1789)
Lagrange (1736-1813)
Laplace (1749-1827)
Monge (1746-1818)
Lazare Carnot (1753-1823)
Sadi Carnot (1796-1832)
Biot (1774-1862)
Legendre (1752-1833)
Poisson (1781-1840)
Euler (1707-1783)
Ampere (1775-1836)

Work on the theory of heat
"On the Propagation of Heat in Solid Bodies"

- 234 pages of book, the Institut de France in Paris -

- Read to the Paris Institute on December 21st, 1807
- 1804(36) – 1807(39) in Grenoble and probably during in Egypt
- Review Committee : Lagrange, Laplace, Monge and Lacroix
- Results: *Highly recognized mathematical analysis of physical phenomena outside the terms of reference of Newton's law of gravitation, but*

  - Lagrange and Laplace in 1808

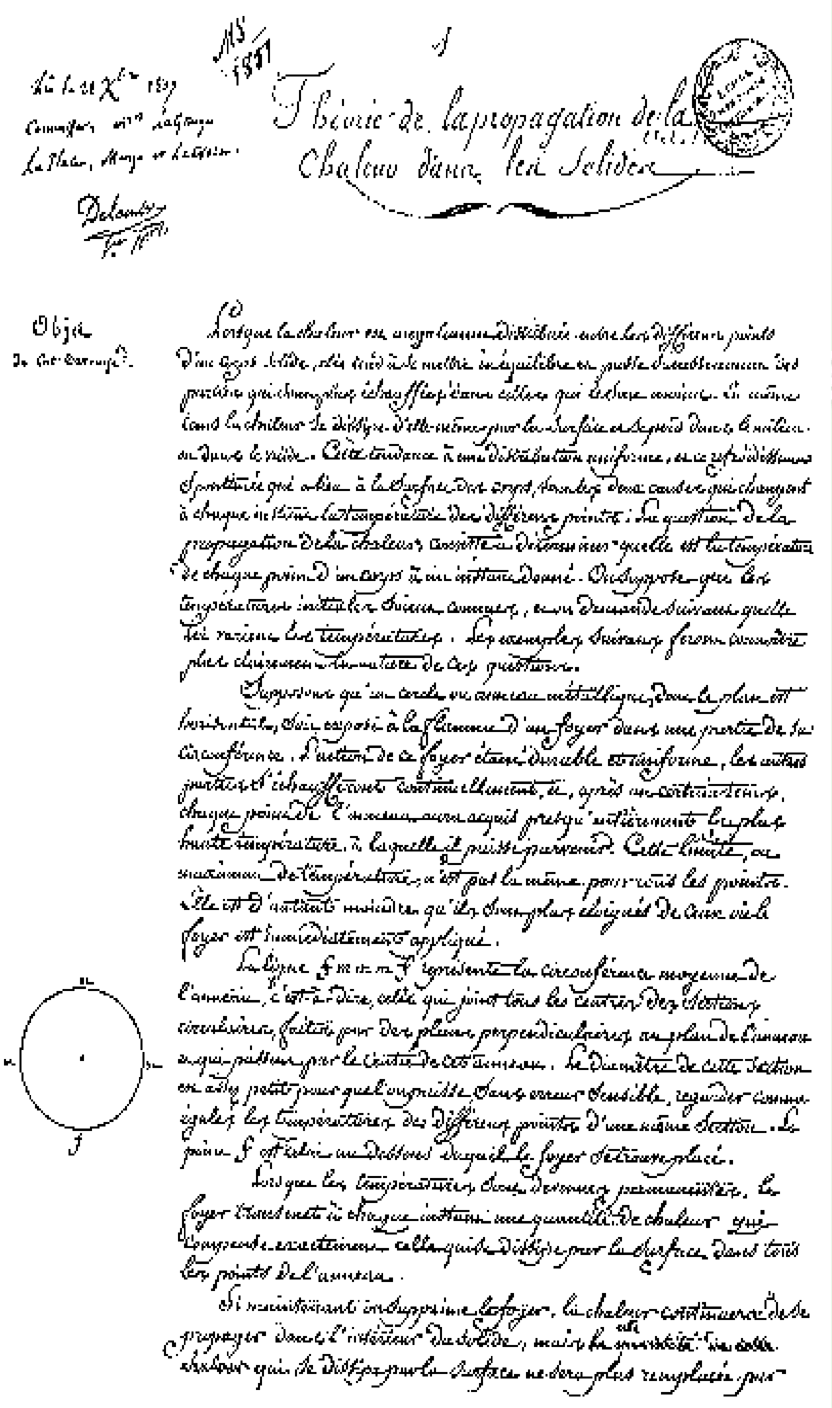
    Fourier's expansion of functions as trigonometric series

  - Biot

    Derivation of the equations of transfer of heat

    Reference to Biot's 1804 paper

Fourier's Original Writing



Théorie de la propagation de la Chaleur dans les Solides

- January 2nd, 1810

The Paris Institute set the 1811 mathematics prize on the subject of the propagation of heat in solid bodies to be in by 1811 October 1st :

"Give the mathematical theory of heat and compare the result of this theory with exact experiments."

Fourier submitted the 1807 memoir with additional work on the cooling of infinite solids and terrestrial and radiant heat

Award(Examiners) Committee:
- Lagrange, Laplace, Malus, Haüy and Legendre

“This work contains the true differential equations of the transmission of heat, both in the interior of the bodies and at their surface and the novelty of the crown this work, observing, however, that the manner of arriving at its equations is not free from difficulties and its analysis of integration still leaves something to be desired, both relative to its generality and on the side of rigour.”

“...The author of this paper is the Baron Fourier, Member of Legion of Honour, Baron of the Empire.”

The prize was **awarded to Fourier**, but with criticism : Good work to be crowned to **fit the class of the Institute**, but something further needed on the score of generality and rigor. No publication in the journals of the Institute.

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Napoleon's Exile
Not through Grenoble

Exile to Elba (1814)

Napoleon’s Return through Grenoble

Napoleon’s triumph return from Elba

- 1815(47) Appointed as Prefect of the Rhone centered at Lyon, but resigned soon before the end of Napoleon's Hundred Days and moved to Paris to follow his intellectual life and get his prize paper and book printed
- 1817(49) Elected to the Académie des Sciences
- 1822(54) Secretaire perpetual to the Académie des Sciences to succeed Delambre.

  Published Fourier's prize winning essay "Theorie analytique de la chaleur"

  Delambre arranged the publication of Fourier's work before his death and Fourier's prize winning essay was published.

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ERRATA.

Fourier's Ring

CHAPTER IV.

OF THE LINEAR AND VARIED MOVEMENT OF HEAT IN A RING.

SECTION I.

General solution of the problem.

238. THE equation which expresses the movement of heat in a ring has been stated in Article 105; it is

$$\frac{dv}{dt} = \frac{K}{CD}\frac{d^2v}{dx^2} - \frac{hl}{CDS}v \text{ ....} (b).$$

The problem is now to integrate this equation: we may write it simply

$$\frac{dv}{dt} = k\frac{d^2v}{dx^2} - hv,$$

wherein k represents $\frac{K}{CD}$, and h represents $\frac{hl}{CDS}$, x denotes the length of the arc included between a point m of the ring and the origin O, and v is the temperature which would be observed at the point m after a given time t. We first assume $v = e^{-ht}u$, u being a new unknown, whence we deduce $\frac{du}{dt} = k\frac{d^2u}{dx^2}$; now this equation belongs to the case in which the radiation is nul at the surface, since it may be derived from the preceding equation by making $h = 0$: we conclude from it that the different points of the ring are cooled successively, by the action of the medium, without this circumstance disturbing in any manner the law of the distribution of the heat.

In fact on integrating the equation $\frac{du}{dt} = k\frac{d^2u}{dx^2}$, we should find the values of u which correspond to different points of the

From Carslaw and Jaeger, P.160(1973)

One of the simplest and most suggestive problems in the conduction of heat, when the temperature depends only upon one coordinate and the time, is Fourier's problem of the ring. This problem is also of special interest, as it was the first to which Fourier applied his mathematical theory, and for which the results of his mathematical investigations were compared with the facts of experiment.†

The ring consists of a small cross-section twisted into a circle (or other closed curve). Then with the notation and assumptions of § 4.2 the differential equation for the temperature in the ring is 4.2 (4), that is

$$\frac{\partial v}{\partial t} = \kappa \frac{\partial^2 v}{\partial x^2} - \nu v. \tag{1}$$

We suppose the length of the ring to be $2l$, so that taking the origin at any convenient point we have to solve (1) in the region $-l \leqslant x \leqslant l$. Since the ring forms a closed curve we do not have boundary conditions at $x = \pm l$, but instead the condition that v is to be periodic with period $2l$ in x, that is

$$v(x,t) = v(x+2nl,t), \quad n = 1, 2, \dots . \tag{2}$$

I. *Initial temperature* $f(x)$. *No radiation*

We assume that $f(x)$ can be expanded in the Fourier series

$$f(x) = \sum_{n=0}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}. \tag{3}$$

Then
$$v = \sum_{n=0}^{\infty} a_n e^{-\kappa n^2\pi^2 t/l^2} \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n e^{-\kappa n^2\pi^2 t/l^2} \sin\frac{n\pi x}{l} \tag{4}$$

satisfies all the conditions of the problem. This may be verified‡ as in § 3.3.

The solution for the case of radiation follows on substituting $v = ue^{-\nu t}$ in (1).

† Fourier, *Théorie analytique de la chaleur*, Chaps. II and IV.

‡ It may also be verified that in this case v and $\partial v/\partial x$ are continuous at $x = \pm l$ for $t > 0$, as they should be since the ring forms a continuous curve. They need not be continuous there when $t = 0$.

§ Fourier, loc. cit., §§ 107–10.

What made Fourier's interest and motivation in heat propagation?

- Grenoble and Egypt (?)
- In 1736, Academy of Science of Paris had proposed "the Study of the nature and the Propagation of Fire" as the subject of a prize essay. Euler was crowned with two others.
- Napoleon favored the mathematical sciences and created prizes for physical discoveries.
- Earlier Work
- The French Revolution

- Lagrange's Memoir on "The Nature and Propagation of Sound" (1759)

$$y = 2\int_0^1 \sum_1^\infty (\sin n\pi x' \sin n\pi x \cos n\pi at) f(x')dx' + \frac{2}{a\pi}\int_0^1 \sum_1^\infty \frac{1}{n}(\sin n\pi x' \sin n\pi x \sin n\pi at) F(x')dx'$$

where, f(x) : initial displacement, F(x) : initial velocity

at t=0 $\quad f(x) = 2\sum_1^\infty A_n \sin n\pi x$

$$A_n = \int_0^1 \sin n\pi x' f(x')dx'$$

- Why Lagrange missed?

The object of Lagrange was to obtain the functional solution, not the coefficients!

- The French Revolution (Described by G. Cuvier (1769-1832))
  - Reconstruction with demolition
  - Practical popularization of science and to establish its educational and technical importance
  - The Memoirs of the Academy: confined to the measured and concise statements of facts or to theories capable of mathematical verification and treatment
  - Defense and Patriotism:
    . L. Carnot and many other mathematicians and scientists
    . New methods of manufacturing, natural resources
    . Existing academics and colleges: organized a system of public instruction
    . Professors and officers
    . A great number of students studied the different branches of knowledge and the art of teaching under the greatest masters

- In the 19th century

  - The revolutionary transformation of the traditional scientific disciplines into the exact sciences : mathematization of sciences, electricity, magnetism, mechanics, light, **heat**

  - Method of approaches to formulate :
    . Facts and underlying causes
    . Facts and observations

- Publications by Fourier, 1820-1829

Light & Wave Motion	2
Heat	5
Mathematics and Mechanics	16
Total	23

Among 295 paper published by 14 scientists including Laplace, Fourier, Arago, Biot, Poisson, Ampere, Dulong, etc. in the period of 1820-1829, 30 papers are related to Heat (10%).

Fourier

- Theoretical and experimental physicist
- Mathematician
- Theorie Analytique de la Chaleur
  - On December 21st of 1807, 234-page work
  - On 1822, 433 articles in 541 pages

The Theory of heat

- 1807 “On the Propagation of Heat in Solid Bodies”
- 1822 “Theorie analytique de la chaleur”
- 1824 Sadi Carnot
- 1840 James Prescott Joule (1818-1889)
- 1842 Julius Robert von Mayer (1814-1878)

Fourier

- Elegant writer
- Master of good style
- Almost no grammatical flaws

"To found the theory, it was in the first place necessary to distinguish and define with precision the elementary properties which determine the action of heat. I then perceived that all the phenomena which depend on this action resolve themselves into a very small number of general and simple facts; whereby every physical problem of this kind is brought back to an investigation of mathematical analysis. From these general facts I have concluded that to determine numerically the most varied movements of heat, it is sufficient to submit each substance to three fundamental observations. Different bodies in fact do not possess in the same degree the power to *contain* heat, *to receive or transmit it across their surfaces*, nor to *conduct* it through the interior of their masses. These are three specific qualities which our theory clearly distinguishes and shows how to measure."

Joseph Fourier, 1822

- Heat Propagation

"But whatever may be the range of mechanical theories, they do not apply to the effects of heat. These make up a special order of phenomena, which cannot be explained by the principles of motion and equilibrium. We have for a long time been in possession of ingenious instruments adapted to measure many of these effects; valuable observations have been collected; but in this manner partial results only have become known, and not the mathematical demonstration of the laws which include them all."

0. All motion of heat depends on temperature differences
1. Power of bodies to contain heat
2. Power of bodies to receive or transmit heat across their surfaces
3. Power to conduct heat through the interior of their masses

Fourier’s achievements are

- Outside the scope of rational and celestial mechanics
- Theory of functions and representation as trigonometric series
- Mathematical analysis of physical phenomena
- Novel treatment and application of linear differential equations to nontrivial boundary value problems with separable spatial and temporal variables
- To distinguish between two kinds of physical behavior – action at an interior point and action on a surface boundary
- Equations in a coordinate system appropriate to the problem
- Explicit statements of initial conditions

Unfavorable receptions

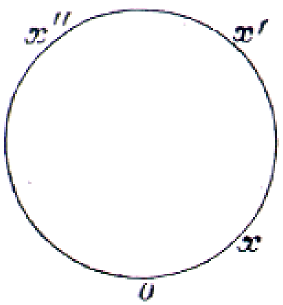
- Rigorous proof for convergence
- Lagrange’s and Euler’s earlier work
- Scientific rivals
- Isolation from Paris and no regular intellectual contact
- Political and administrative duty

Fourier's Experimental Work

- Conducted experiments in the period of 1806-1807
- In his 1807 paper,
  - Steady thermal state in annulus
  - Heat diffusion in annulus
  - Heat diffusion in sphere
  - Comparison between sphere and cube on the rate of cooling
  - Error and response of thermometers
- Mercury thermometer: 0^{o}R(Réaumur scale) – 80^{o}R
- Heating with Argand lamp
- Time : 3 different clocks – 9h21m, 9h21m, 9h 20m
- Room temperature : 19^{o}R or 20^{o}R

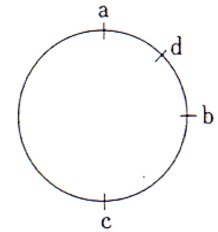
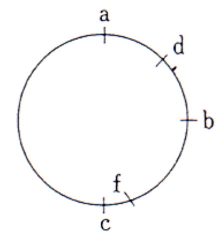
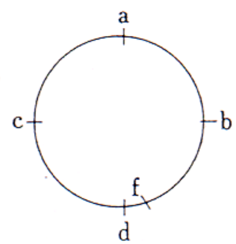
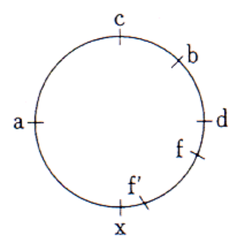
the ring (see figure 3), l is the perimeter of the section whose area is S, the coefficient h measures the external conducibility, K the internal conducibility, C the specific capacity for heat, D the density. The line $oxx'x''$ represents the mean circumference of the armlet, or that line which passes through the centres of figure of all the sections; the distance of a section from the origin o is measured by the arc whose length is x; R is the radius of the mean circumference.

Fig. 3.



It is supposed that on account of the small dimensions and of the form of the section, we may consider the temperature at the different points of the same section to be equal.

Table I. Experiments on the steady state in annulus.

	Date	Time*	Disposition of heat sources and thermometers	Observed temperatures	Other remarks	Comparative references
1	1806 Summer	6 h 17 min		θa 45°1/4 θb = 67°4/5 θd = 51°2/3 θr = 17°1/2	Calculation on the values of h/K and K. f was near to c. θc was omitted for Th c unclibrated.	MS. 22526, f. 35r·36r
2	1806 7/31	5 h 22 min		θa = 44° θb = 66° θd = 50°7/12 θc = 99°1/3 θr = 17°2/3	Calculation on the value of H/K. Argand lamp was used.	1807 paper, Art. 160 MS. 22526, f.3r·4v·5 6·7·8·9·10 11·12·17r
3	1806 9/ 3	10 h 32 min (6 h 39 min)		θa = 41° θb = 60°1/7 θd = 100°1/3 θc = 50° θr = 17°	After removing f, heat diffusion was observed. (for the accident of a lamp, observation was prolonged.)	MS. 2256, f. 24r·26rv
4	1806 9/30	5 h 35 min		θa = 53°1/2 θb = 68°1/4 θc = 52°5/6 θd = 100°1/3 θx = 100° θr = 16°	At the same time, heat sources f, f' was used.	MS. 22526, f. 37·38·39 40·41r.

θa, θb, θc, θd, θx and θr: indications of thermometers a, b, c, d, x and temperature of air on Réaumur's division.

f and f': heat sources. H and K: external and internal conductivity. Th: thermometer.

* time required from the start of heating to the end of observation.

After Fourier

- **Fourier Series**
  - **Poisson**
  - **Cauchy**
  - **Dirichlet**
  - **Riemann**

- **Fourier's law**
  - **Ohm's law (1826)** $I = \frac{1}{R} V$
  - **Fick's law (1855)** $\dot{m} = -D \frac{\partial C}{\partial x}$
  - **Kelvin**

Conclusion

- Fourier
- Grenoble

Jean Baptiste Joseph Fourier