Some Problems in Radiative Transport

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LECTURE

ABSTRACT

Interest in satellites, plasmas, rockets, ramjets and reentry from space has swelled the trickle of engineering research on radiative transport, motivated primarily by an interest in furnaces, to a healthy stream motivated by problems of ever-increasing breadth. The rapid growth of the field has been accompanied, as expected, by often uncoordinated, sometimes overlapping or repetitive research on a problem, and by the use of nomenclature and tools reflecting the wide differences in background of the investigators. Under such circumstances frequent stock-taking is desirable. It is proposed here to review some of the work of the recent past, but not thoroughly; to comment on some of the problems and report on work in progress, and to indicate desirable fields of exploration. In particular, emphasis will be put on problems of interaction of radiation and other mechanisms, of gas radiation with temperature gradients, of transition from flux responsive to local conditions (diffusion) to action-at-a-distance.

NOMENCLATURE

(not including terms adequately identified in text)

A — Area of a surface, $(\text{lgth})^2$.

a, a' — Weighting factor, dimensionless. Prime indicates use of emissivity, no prime for absorptivity. Temperature dependence indicated by $a(T)$. Subscript n indicates member of class.

B — Characteristic dimension of a zone, (lgth).

b — Ratio, absorption coefficient to total extinction (absorption plus scatter) coefficient, $k_a/(k_a+k_s)$; also exponent on temperature.

C_n — A constant in equation 4, dimensions T^b.

c — Velocity of light.

E — Hemispherical black-body emissive power, σT^4, (energy/area time temp^4), where T is local "molecular" temperature. Prime indicates gas. Subscripts G, GM, S: gas, mean-gas, surface.

$\overline{E_n(x)}$ — The n^{th} exponential integral of x, available in tables.

$\overline{GG}$ — Total interchange area between any two gas zones, allowing for multiple scatter at all gas zones and multiple reflection at all surfaces, $(\text{length})^2$.

gg — Direct interchange area between 2 gas zones, $(\text{lgth})^2$. The quantity by which $(E_{G_1} - E_{G_2})$ is multiplied to give rate of *direct* energy transfer by radiation.

$\overrightarrow{GG}$ — Directed total-interchange area; $\overline{GG}$ after allowing for effect of temperature on radiating characteristics of that zone at root of arrow, and after summing over the n gray gas components present.

$\overline{GS}$ — Like GG, except for reference to gas-surface interchange.

$\overline{gs}$	Like gg, except for reference to gas-surface interchange.
$\overrightarrow{GS}$	Like $\overrightarrow{GG}$, except for reference to gas-surface interchange.
K	Absorption coefficient or total extinction coefficient, $(\text{lgth})^{-1}$.
k	Absorption coefficient based on partial pressure, $(\text{atm ft})^{-1}$. $kP = K$.
L	System dimension, (lgth).
l_s	Radiation mean free path, (lgth), $= 1/K$. The distance a collimated beam travels during attenuation to e^{-1} of the original intensity.
(Nu)	Nusselt number, hm/λ, where m is mean hydraulic radius ($2L$ for slabs). Subscripts T for total, due to $\lambda_c + \lambda_r$.
P	Partial pressure of radiating components, as $P_{CO_2 + H_2O}$, (atm).
$\vec{q}$	Flux, (energy)/(area)(time), across plane of particular orientation. Subscript r for radiant.
q/A	Flux per unit area, (energy)/(area)(time), as applied to surfaces.
$\overline{SS}$	Like $\overline{GG}$, except for reference to surface-surface interchange.
$\overline{ss}$	Like $\overline{gg}$, except for reference to surface-surface interchange.
$\overrightarrow{SS}$	Like $\overrightarrow{GG}$, except for reference to surface-surface interchange.
T	Temperature. Subscripts G (gas), S surface.
V	Volume.
W_i	Leaving-flux density (hemispherical) at surface A_i, (energy)/(time)(area). Includes emission plus reflection.
W_i'	So defined that $4KW_i'$ = total rate of energy emission plus scatter by V_i, (energy)/(time)(area).
${}_jW_i$, ${}_jW_i'$	Dimensionless value of W_i and W_i' when j is the sole emitter in the system and the temperature of j is such that its black-body emissive power is 1.
x	Distance.
α	Absorptivity, dimensionless. Subscript GS refers to absorber and source of radiation in sequence.
δ_{ij}	The Kronecker delta, having value zero except when $i = j$, when its value is 1.
ϵ	Emissivity, dimensionless. Subscripts G, S, i refer to gas, surface, i^{th} surface.
λ	Thermal conductivity, (energy)/(time) (temp diff) (lgth)
σ	Stefan-Boltzmann constant, (energy)/(area)(time)$(\text{temp})^4$.
τ	As used by Konakov, the "radiating" temperature. Related to W by $\sigma\tau^4 = W'$.

*For non-gaseous emitters with a refractive index n significantly different from 1, E is $n^2 E_{\text{vac.}}$, and an n^2 belongs inside the last parenthesis.

1. RADIATIVE TRANSPORT AS A DIFFUSION PROCESS

Consider a unidirectional temperature field in an emitting-absorbing medium, and consider an isothermal plane located a great distance – measured in radiation mean free paths – from bounding walls. Since radiative flux depends not on absolute values of emissive power $E (\equiv \sigma T^4)$ but on differences, it is clear that if the gradient in E is constant for some distance on either side of the plane of interest, the radiative flux normal to the plane must be proportional to the gradient and to the mean free path l_s, which is the reciprocal of the absorption coefficient K. This concept must be many decades old. Rosseland presented a derivation in 1931 [2]. Kellet in 1952 approximated the unidirectional flux in glass [3], but without consideration of the 2π distribution of direction of the radiant beams crossing the principal plane. Czerny and Genzel [4] and later others [8, 14, 15, 17] derived the relation

$$\vec{q}_{\text{rad}} = -\frac{4}{3K}\,\text{grad}\,E = -\left(\frac{16\,\sigma T^3}{3K}\right)\frac{dT}{dx} \qquad (1)^*$$

This attractive limiting law, which permits treatment of radiative flux with a differential rather than an integral equation, has begun to see much use and some misuse; and its limits of applicability merit consideration.

The validity of the coefficient $4/3K$ has been put in question by three authors recently. Kolchenogova and Shorin [19] have used the value $4/m^2K$, with m^2 dependent on the spatial distribution of intensity and numerically 4, despite Shorin's use of $4/3K$ in a paper two years earlier [13]. More recently Konakov [30] has presented a derivation which leads to the value $1/K$, in agreement with Kolchenogova and Shorin. Konakov follows the established argument that, analogous to conduction,

$$\vec{q}_{\text{rad}} = -D\,\text{grad}\,\phi$$

where D is the diffusivity of photons, proportional to the product of their velocity C and mean free path l_s, and ϕ is the radiation density of local space, $4E/c$; but for diffusivity he uses $1/4\,cl_s$. The product D grad ϕ is then l_s grad E or $(1/K)$ grad E. From the sketch he presents it appears that his factor of 1/4 must have been derived somewhat as follows:

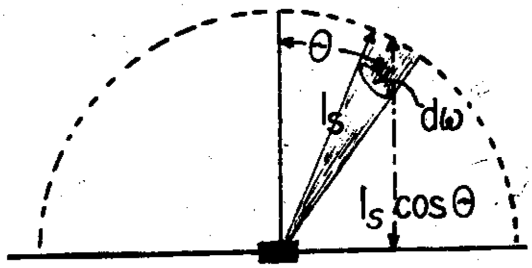

FIG. 1 – THE DIFFUSION OF PHOTONS

Consider emission from a unit volume lying in the zero plane (see Fig. 1). Equal chance of emission in all directions through 2π steradians gives an average distance traveled of

$$\int_{2\pi} (l_s \cos\theta)\, d\omega/2\pi,$$

$$\text{or} \int_0^{\pi/2} 2\pi l_s \sin\theta \cos\theta\, d\theta/2\pi = \frac{l_s}{2} \;,$$

and since half the photons go the other way, $l_s/4$. Attention should have been focused, however, on all photons passing through the base plane rather than on those originating in a slab located at the base plane, as a result of which different areas of the receiving hemisphere would have a chance of receiving a photon which is measured by $\cos\theta$. Perhaps the reverse process, which in the limit should be the equivalent of the forward one, is easier to visualize; the photons passing through equal areas of emission from the hemispherical surface toward the plane element have chances of hitting it which are proportional to $\cos\theta$. Then the mean forward distance traveled in one mean-free-path of movement is

$$\int_{2\pi} (l_s \cos\theta) \cos\theta\, d\omega/2\pi \;,$$

$$\text{or} \int_0^{\pi/2} 2\pi l_s \sin\theta \cos^2\theta\, d\theta/2\pi = (2/3)\, l_s$$

Allowing for half the photons going the other way, the diffusivity is $c\,l_s/3$ or $c/3K$, and multiplication by grad ϕ gives a result in agreement with (1)*. Filippov's perhaps more satisfying derivation [17], yielding the same result, sets up the flux equations between parallel plates and takes the limit of interior flux as the distance to the plates, measured in mean free paths, becomes great.

Equation 1 is rigorous when grad E is constant. When its constancy does not extend for more than three mean free paths on either side of the plane of interest, use of (1) can produce large error. The attractive simplicity of the relation has in the author's opinion caused it to be used unjustifiably. Konakov, for example, has recently claimed that conduction, convection and radiation in combination can be handled, without integral equations, by judicious use of the radiative diffusion process. He presents relations purporting to permit ready calculation of combined radiation and conduction in systems consisting of gray parallel slabs, concentric cylinders, and concentric spheres bounding a stationary conducting and emitting-absorbing medium; and recommends equations for use over the full range of separating distances between walls, from zero up. Of the 154 equations presented, those covering the case of radiative flux between parallel plates were chosen, for easy comparison with available rigorous solutions. The relations given are:

$$\text{When } L > 2\,l_s,\; \vec{q} = \left\{ \begin{array}{c} \lambda K \dfrac{(\tau_1 - \tau_2)}{KL - 2} + \sigma \dfrac{\tau_1^4 - \tau_2^4}{KL - 2} \\ \text{and} \\ \lambda K\,(T_1 - \tau_1) + \sigma \dfrac{(T_1^4 - \tau_1^4)}{(1/\epsilon_1) - (1/2)} \\ \text{and} \\ \lambda K(\tau_2 - T_2) + \sigma \dfrac{(\tau_2^4 - T_2^4)}{(1/\epsilon_2) - (1/2)} \end{array} \right\} \begin{array}{c} \text{3 equations} \\ \text{in} \\ \vec{q}, \tau_1, \tau_2 \end{array} \qquad (2)$$

$$\text{When } L < 2\,l_s,\; \vec{q} = \lambda K \frac{T_1 - T_2}{2} + \sigma \frac{(T_1^4 - T_2^4)}{\dfrac{1}{\epsilon_1} + \dfrac{1}{\epsilon_2} - 1} \qquad (3)$$

Here L is the wall spacing, λ the thermal conductivity, and τ_1 and τ_2 are intermediate "radiation temperatures" which are eliminated in solution for $\vec{q}$. Figure 2 gives the numerical

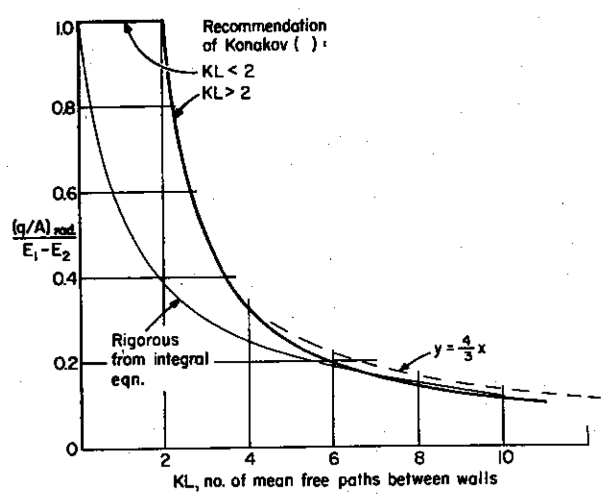

FIG. 2 – RADIATIVE FLUX THROUGH GAS BETWEEN HOT AND COLD WALLS. COMPARISON OF KONAKOV'S RECOMMENDATIONS [30] WITH RIGOROUS SOLUTION.

*Modification of the diffusion derivation to allow for refractive index n: change c to c/n and E to $E_{vac}n^2$.

consequence of these relations, heavy lines, plotted as $\vec{q}_r/(E_1 - E_2)$ vs KL, for the black-wall case with no conduction. For comparison, or contrast, the rigorous solution to this problem is presented as the light line. This is based on determination of the temperature field ϕ $(\equiv(E_G - E_1)/(E_2 - E_1))$ from solution of the integral equation

$$\int_0^{Kx_0} \phi(y)\, d\overline{E_2(y_0 - y)} - \int_{Kx_0}^{KL} \phi(y)\, d\overline{E_2(y - y_0)} + \overline{E_2(KL - y_0)} = 2\phi(y_0),$$

and insertion into the flux equation

$$\frac{(q/A)_{\text{rad}}}{E_2 - E_1} = 2\int_0^{KL} \phi(y)\, d\,\overline{E_3(y)} + 2\overline{E_3(KL)},$$ where

$\overline{E_2(\;)}$ and $\overline{E_3(\;)}$ are the second and third exponential integrals.*

The relations given for cylinders and spheres *(l.c.)* are of a construction similar to (2) and (3). It is clear that these relations are not generally valid, and the radiative flux is not representable by (1) except where E extends with a constant gradient for several mean free paths. Many problems of practical interest lie in the range of 0.1 to 2 mean free paths dimension in the direction of the gradient in E.

A third line, dashed, appears on Fig. 2. It represents direct use of (1) by evaluation of grad E simply as $(E_2 - E_1)/L$. It ultimately merges with the line representing the rigorous solution, and the two differ from the heavy line, in the limit, in the ratio 4/3.

2. TRANSITION FROM DIFFUSION TO ACTION-AT-A-DISTANCE

The above discussion leads to the conclusion that radiative transport may be divided into three regimes, quite different in the mathematical problems to which they give rise.

(1) At extremely high values of optical depth the radiation emitted from a volume element suffers such rapid attenuation that the net rate of energy loss from it is dependent on local conditions, — the gradient in the emissive-power field. This diffusion process has already been discussed in connection with (1). It was seen to be mathematically equivalent to gaseous conduction except that T^4 is the potential function rather than T. If conduction and radiation are combined in this regime, one of them can be linearized in the other and they can then be added.

(2) At intermediate values of optical depth every part of the system influences every other part which it can "see"; and by "seeing" is meant seeing either directly or in the diffuse mirror of any non-black walls or by scatter processes involving components in the gas. There is "action-at-a-distance", and integral equations must be solved.

(3) At extremely low values of optical depth of the system, measured in free paths, the radiation that is emitted throughout the volume passes without attenuation to an absorbing surface or through the boundary of the system. Radiative flux at any plane, representing the sum of the contributions of all volumes and surfaces to it, varies with the position of the plane; and the gradient, either in temperature or in black-body emissive power, is in general not constant. But the net flux from any volume, by all mechanisms, is dependent solely on its temperature and conditions at its own boundaries, and action-at-a-distance may be said to be absent.

An example in which the three regimes stand out clearly comes from consideration of the same slab problem discussed above, the case of stagnant gas in thermal equilibrium between hot and cold parallel black walls. Although Fig. 2 presented results on the problem of total interchange between walls, more illuminating are the results on net interchange between the hot wall and the gas, or the gas and the cold wall. Fig. 3 presents these, calculated by the zoning technique. Substantially the whole of the range of abscissa KL lies in regime 2 where use of the integral equation is necessary. When KL is large, however, the transport process approaches one of diffusion, and (1) is applicable (dotted line marked $y = (4/3)\,x$); this is regime 1. When KL is very small, radiation from the gas slab passes unattenuated to the walls; and since every element of gas sees the two walls equally well, all elements are at the same temperature corresponding to the arithmetic mean emissive power $(E_{s_1} + E_{s_2})/2$. An isothermal gas slab has an emissivity approaching $2KL$ in the limit as KL approaches zero [1]. Combination of these concepts gives

$$q_{G \rightleftharpoons s_1}/A = \left(\frac{E_{s_1} + E_{s_2}}{2} - E_{s_1}\right) 2KL = (E_{s_2} - E_{s_1})KL$$

This represents performance in regime 3, indicated on the diagram by the dotted line marked $y = x$. Plainly, regime 3 for this problem is in a range of KL below 0.1.

Perhaps with enough cases studied over the range of all 3 regimes to show how far the rigorous treatment differs numerically from either of the two limit approximations, the engineer will be in a position to

*The reader is cautioned not to confuse $\overline{E_n(\;)}$ with E_n, the emissive power of a surface.

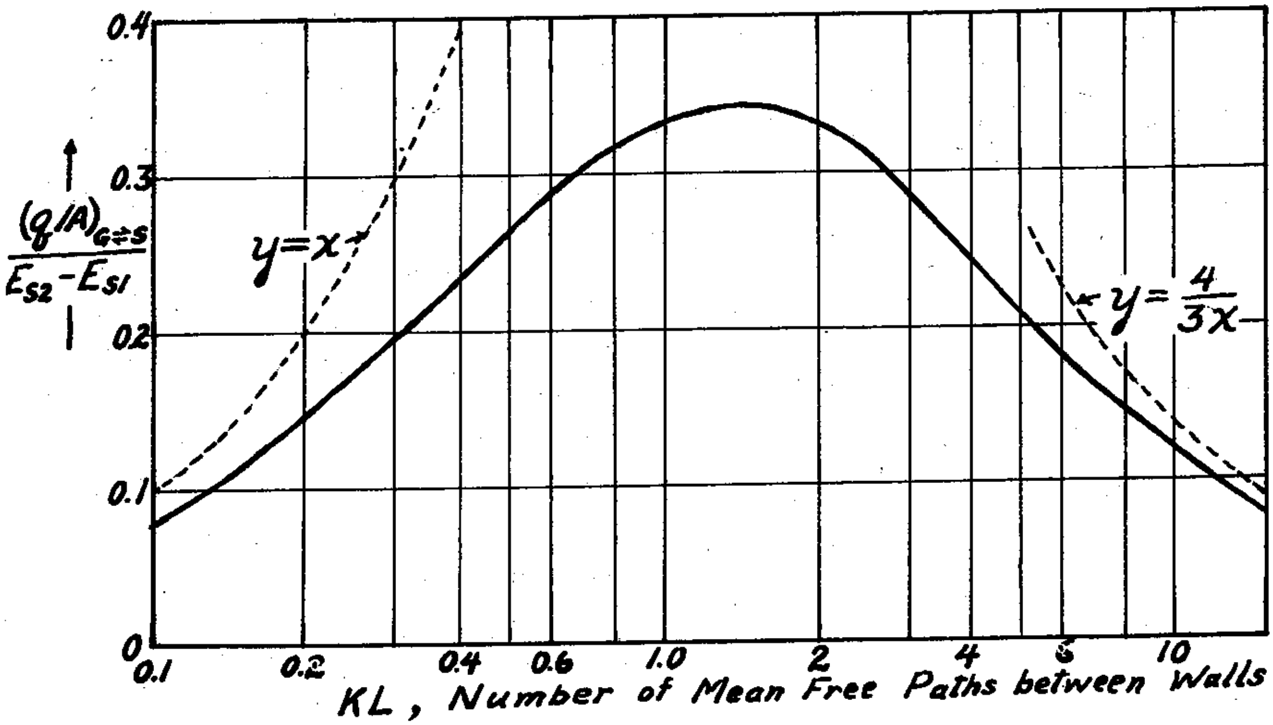

FIG. 3 – THE THREE REGIMES OF RADIATIVE TRANSPORT. ILLUSTRATION: GRAY GAS CONFINED BETWEEN HOT AND COLD PARALLEL BLACK WALLS. FLUX FROM GAS TO COLD WALL.

estimate answers to many of his problems adequately by use of the relatively simple mathematics of regime 1 or 3. A note of pessimism must be added here, however. The difference between real gases and gray gases to be referred to in discussing (7), – the tendency of different absorption bands to dominate in different ranges of PL – keeps the value $K_{av}L$ from changing in proportion to L; and one has difficulty in moving a real-gas problem very far from the middle range of Fig. 3. For the CO_2-H_2O system studied, when P was kept constant at 0.25 atm and L was varied from 0.5 ft to 8 ft, $K_{av}L$ varied only from 0.8 to 1.6.

3. INTERACTION OF RADIATION AND OTHER TRANSPORT MECHANISMS

The interaction of radiation, convection, and conduction, – the modification of the temperature field and sometimes of the flow field due solely to one mechanism by the presence of the others, – is in general a problem of considerable complexity. Some of the work done in this area will be reviewed briefly, and a relatively simple interaction problem involving convection will be considered in some detail.

Walther, Dorr, and Eller [10] have followed the work of Genzel and others [4, 5, 6, 8, 9] on unidirectional radiative flux through glass slabs with a study of steady-state temperature profiles in a gray slab with and without conduction. (Because of failure to allow for the complicating effects of refractive index at the slab boundaries, the method presented is valid for gas slabs with refractive index near 1, but somewhat in error for glass.) The finite difference in temperature of a wall and of the medium touching it, which generally characterizes radiation, was found to disappear under the influence of conduction, as expected. (Discontinuity at the walls is of course also associated with conduction, but it is not significant in a system many conduction-mean-free-paths thick. The ratio l_{rad}/l_{cond} is generally a number of many orders of magnitude).

Gardon [22], in an important paper on heat flux in glass, allowed for the change in refractive index at the boundaries, and for multiple reflection, and included conduction.

Adrianov and Shorin [21] treated the case of a gray gas flowing through a cylindrical black-walled duct of uniform wall temperature. The gas was assumed to enter at a uniform temperature, and the limiting cases of plug flow and parabolic velocity profile were considered. This would appear to constitute a study of interaction, at least to the extent that the velocity profile should modify the radiation field; and the work was labeled a "general solution" of the problem of radiative transfer in the flow of a uniform medium. But in the analysis each gas element or shell was allowed to cool as though it saw cold walls through a medium (the other shells) with which it did *not* exchange heat, even though allowance *was* made for attenuation of its emitted energy by absorption in passage through the other shells to the walls. This pair of assumptions has no counterpart in reality. The effect of axial temperature gradients was also ignored, but that assumption can have its physical counterpart in long systems. Experiments on hot dust-laden combustion gases were carried out on a 10 cm cylinder 80 cm long, presumably to confirm the analysis of the radiation problem. A rough calculation by the author indicates that the empirical equation fitted to the data predicts total heat transfer coefficients (radiation plus convection) of the order of ten percent greater than those expected from convection alone.

R. and M. Goulard [26] treated the case of a plane layer of stagnant gas in thermal steady state between a gray and a transparent wall, in application to air at 4000 to 7000K. Allowance was made for interaction between conduction and radiation, but this interaction was greatly simplified by the assumption (probably justifiable for the physical case of interest) that emission from any gas layer passed unattenuated to the walls. The gas was assumed gray.

Among the researches on conduction-radiation interaction to which the reader's attention should be called are the excellent works of Churchill and associates [34] on absorption, scatter, and conduction in powders and fibre mats, and the paper of Kadanoff [35] on radiation, scatter and conduction in ablating bodies.

To generalize most readily about the consequences of interaction between radiation and conduction-convection one needs a simple geometry. Probably the simplest is that of steady flow of gas between infinite parallel black plates maintained at constant but different temperatures. Consider such a system, at a point far enough from entry to make the temperature and velocity profiles invariant. Let the problem be to examine how radiation and convection together modify the action each would exert in the absence of the other, by letting the gas be an emitter, — a real (non-gray) gas at a temperature high enough to make radiation important.

The net total-energy flux will be constant at all planes parallel to the walls, and the net radiative plus convective flux from hot wall to gas will equal that from gas to cold wall; but the net wall-to-wall flux will include an additional term due to radiation alone.

To minimize the number of dimensionless parameters of the system, the following restrictions will be imposed:

1. The difference in wall surface temperatures $T_{s_2} - T_{s_1}$ is small relative to their mean value $(T_{s_1} + T_{s_2})/2$. Because of the resulting skew-symmetry of the temperature profile, the mean gas temperature $T_{G,M}$ will also equal $(T_{s_1} + T_{s_2})/2$.

2. The gas total emissivity and absorptivity are representable by

$$\epsilon_G = \sum_1^n a'_n(T_G)(1-e^{-K_nL}) \ , \ \sum_0^n a'_n = 1$$

$$\alpha_{GS} = \sum_1^n a_n(T_s)(1-e^{-K_nL}) \ , \ \sum_0^n a_n = 1$$

In words, emissivity is a weighted sum of the emissivities $1-e^{-K_nL}$ of n different gray gases, weighted in proportion to the factor a'_n, which is a function of gas temperature. In addition there is a clear-gas ($K=0$) component with weighting factor a_0, which does not enter the formulation of ϵ_G but does affect τ_G^0, the transmittance of the gas. The K's are not a function of temperature. Absorptivity, by the gas, of radiation from a surface is built up the same way, the weighting factors a_n being functions of the temperature of the emitting surface T_s^n and the K's being the same as those used for emissivity.

These relations can sometimes be used with n only 1; i.e., a mixture of one gray plus a clear gas is assumed. Over a wider range of path lengths, two or three gray-gas terms may be needed. In a test on a $CO_2H_2O\text{-}N_2$ mixture, $n = 3$ produced equations in agreement, within 5%, with emissivity and absorptivity data over a 2500-fold range of $(p_{CO_2} + p_{H_2O})L$ and over the temperature range 1500-3500R [29, 33].

3. The temperature variation of emissivity is given by

$$\left[a'_n(T_G)\right] T_G^b = \left[a_n(T_s)\right] T_s^b = C_n \qquad (4)$$

The values a' and a must be the same when $T_G = T_s$, from the second-law, and the assumption of a simple power law must be adequate if the temperature range is small. For CO_2-H_2O it is quite satisfactory over a 1000 R range. Plainly it can produce an absurdity if pushed too far, since $\sum_1^n a_n > 1$ has no physical reality.

With these restrictions a radiation model law can be obtained, giving the interchange between one wall and the gas, in a form suitable for combining with convection.

$$(q/A)_r = f_1(\epsilon_s, E_{GM}, E_s, a'_1(T_G), a_1(T_s), a'_n/a_1, a_n/a_1, K_1L, K_n/K_1)$$

For a specific mixture the K_n/K_1's are constant and the a_n/a_1's and a'_n/a'_1's are approximately so, and

$$(q/A)_r = f_2([a'_1(T_{G,M})]E_{G,M} f_3(K_1L,\epsilon_s), [a_1(T_s)]E_s f_3(K_1L,\epsilon_s))$$

Since this function must be of a form which goes to zero when $E_{G,M}=E_3$ and is proportional to $E_{G,M}$ when $E_s = 0$, it may be written

$$\frac{(q/A)_r}{[a'_1(T_{G,M})]E_{G,M} - [a_1(T_s)]E_s} \approx f_3(K_1L,\epsilon_s) \quad (5)$$

If in the above expression a'_1 and a_1 are substantially the same or if $(E_G/E_s) \gg 1$ or $\ll 1$, $E_G - E_s$ emerges as an argument in the denominator. A more general result comes from use of restriction 3 above (4). Use of it and replacement of E by σT^4 changes the left side of the above relation to

$$\frac{(q/A)_r}{C\sigma\,[T_{G,M}^{4-b} - T_s^{4-b}]}\ .$$

Making use of the small spread in absolute temperature, one may write

$$\sigma(T_{GM}^{4-b} - T_s^{4-b}) = \sigma\Delta(T^{4-b}) = \sigma\Delta\left[(T^4)^{\frac{4-b}{4}}\right]$$

$$= \sigma\frac{4-b}{4}\ \frac{1}{T_{av}^b}\ \Delta(T^4) = \frac{4-b}{4}\ \frac{E_{GM}-E_s}{T_{av}^b}$$

Insertion of these substitutions into 5 gives

$$\frac{(q/A)_r}{\frac{C}{T_{av}^b}\ \frac{4-b}{4}(E_{G,M}-E_s)} \approx f_3(K_1L,\epsilon_s) \quad (6)$$

The CO_2-H_2O mixture of present interest is fitted moderately well by $b = 1$. Since $C/T^b = C/T = a$, and $\epsilon_G = af_4(KL)$, the denominator of the left side of (6) becomes $\epsilon_G/f_4(KL)$. The function f_4 may be merged with f_3 on the right of the equation, to give finally

$$\frac{(q/A)_r}{\frac{3}{4}\epsilon_G(E_{GM}-E_s)} \approx f_5(K_1L,\epsilon_s) \quad (7)$$

The 3/4 factor has been retained as a reminder of the effect of temperature on gas emissivity, and also because the left side, as it stands, becomes 1 in the limit as the walls become black and the gas becomes perfectly stirred; and E_{GM} then represents the uniform gas temperature.

In using (7) for approximate correlation of the effects of variation in system mean temperature and in K_1L, one should evaluate ϵ_G at the mean gas temperature and at the mean beam length [1, 32] of 1.76 L. (Equation 7 cannot be expected to correlate performance of gases of different radiating characteristics, but different gases could be brought *somewhat* closer together by use of some average KL rather than K_1L. K_{av} could for example be obtained from solution of a pair of equations, one for the full mean beam length of interest and one for half that value, expressing emissivity in Beer's law form as a single gray-plus-clear gas. When this is done, $K_{av}L$ for the CO_2-H_2O system is found to vary only one-ninth as much as K_1L when the latter varies from 1/8 to 2, due to the changing relative importance, with change in L, of the different absorption coefficients characterizing different parts of the spectrum of CO_2-H_2O. This underlines the great difference between gray and real gases.)

Radiation in the system of interest has been reduced to an approximate relation among three groups. Convention by itself conforms to the relation

$$\frac{(q/A)_c}{(\lambda/2L)(T_{GM}-T_{s_1})} = (Nu)_c = f(Pr, Re) \quad (8)$$

(The term $2L$ follows the convention of using 4 times the mean hydraulic radius.) Combined radiation and convection will then be expressible in a six-group function which, on replacing $(E_{GM}-E_{s_1})/(T_{GM}-T_{s_1})$ by $4\sigma T_{av}^3$, becomes

$$\frac{(q/A)_r \text{ or } (q/A)_c \text{ or } (q/A)_{total}}{\frac{\lambda}{L}(T_{GM}-T_{s_1}) \text{ or } \frac{3}{4}\epsilon_G(E_{GM}-E_{s_1})} =$$

$$= f_6\left(K_1L,\epsilon_s, Pr, Re, \frac{3\sigma\epsilon_G T_{av}^3}{(\lambda/L)}\right) \quad (9)$$

Use of the last of the ratios on the left and a combination of the first and last groups on the right to form a new last group gives

$$\frac{(q/A)_{total}}{\frac{3}{4}\epsilon_G(E_{GM}-E_{s_1})} = f_7\left(K_1L,\epsilon_s, Pr, Re, \frac{3\sigma\epsilon_G T_{av}^3}{K_1\lambda}\right) \quad (10)$$

If there were no interaction between radiation and convection the total flux could be expressed, by use of (8) in the form

$$\frac{(q/A)_T}{\frac{3}{4}\epsilon_G(E_{GM}-E_{s_1})} = \frac{(q/A)_r}{\frac{3}{4}\epsilon_G(E_{GM}-E_{s_1})} + \frac{\frac{\lambda}{2L}(T_{GM}-T_{s_1})(Nu)_c}{\frac{3}{4}\epsilon_G(E_{GM}-E_{s_1})}$$

or

$$\frac{(q/A)_T}{\frac{3}{4}\epsilon_G(E_{GM}-E_{s_1})} = \frac{(q/A)_r}{\frac{3}{4}\epsilon_G(E_{GM}-E_{s_1})} + \frac{(Nu)_c}{2K_1L}\cdot\frac{1}{\left(\frac{3\sigma\epsilon_G T_{av}^3}{K_1\lambda}\right)} \quad (11)$$

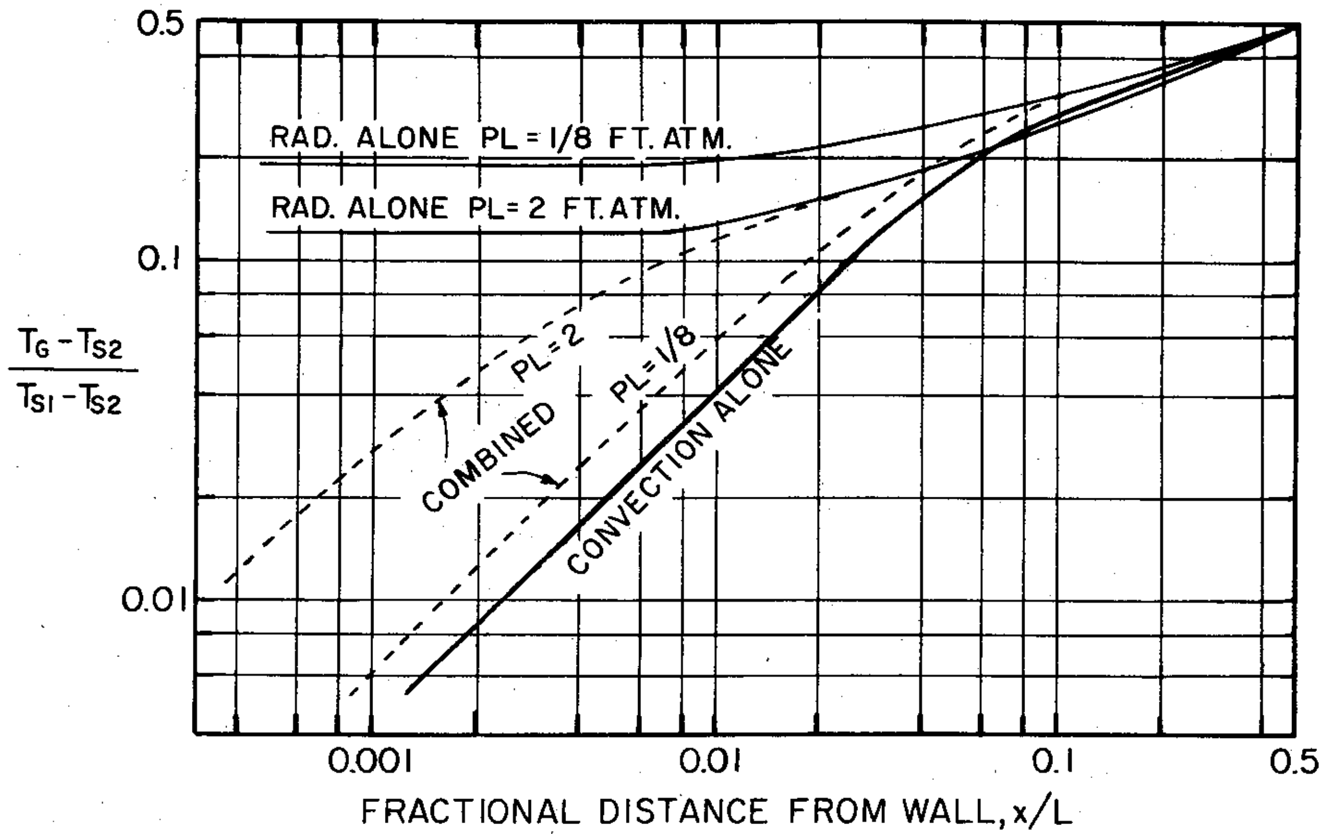

FIG. 4 – EFFECT OF OPTICAL DEPTH, MEASURED BY *PL*, ON THE TEMPERATURE PROFILE THROUGH GAS FLOWING BETWEEN HOT AND COLD BLACK WALLS. Reynolds Number = 5000; mean temperature = 2500R.

Discussion of (10) and (11) will await presentation of some results of work in progress on this interaction problem. Sarofim [33] has studied the case of flow between parallel plates described above, with CO_2-H_2O-N_2 at mean temperatures of 1500 R to 3500 R, values of $P_{CO_2+H_2O}$ of 1/8 to 2 ft atm, and Reynolds Numbers of 5000 to 100,000. Pure convection transport was first calculated from boundary-layer theory to obtain the Nusselt number as a complex function of Reynolds and Prandtl numbers representable empirically by

$$(Nu)_c \equiv \frac{2hL}{\lambda} = 0.019\,(Pr)^{1/3}\,(Re)^{0.8}$$

or, for gas with $Pr = 0.73$, by

$$(Nu)_c = 0.017\,(Re)^{0.8}$$

This is in excellent agreement with experiment [7]. For studying the interaction between radiation and convection, the mixed-gray-gas zone method described below was used, with 54 zones varying 2000-fold in thickness from boundary layer to center.

Figure 4 illustrates the effect, at a Reynolds number of 5000 and a mean temperature of 2500 R, of changing the optical depth of the system, measured by $P_{CO_2+H_2O}L$. The temperature field is expressed as a relation between gas temperature, measured above the cold wall temperature as a base and divided by the total drop across the walls, and the fractional distance x/L from the cold wall. Logarithmic scales are used on both coordinates to emphasize what happens in the boundary layer. The pure radiation curves indicate, as expected, that increasing PL from 1/8 to 2 hides the wall gas more effectively from the center, and its temperature drops. (Temperature discontinuity at the wall is characteristic of radiation). Convection alone produces the expected constant gradient in the laminar sub-layer, which here extends to $x/L = 0.031$. When convection and radiation interact, – dotted lines, – the boundary-layer temperature gradient is no longer constant, since the total flux is constant but the radiation flux varies and therefore $\lambda dT/dx$ must vary. It is to be noted that toward the core the temperature field of the combined processes approaches the pure radiation field, at about 0.02 for $PL = 2$ and 0.1 for $PL = 1/8$. At $Re = 5000$, the combined-action temperature field always lies between the fields of the separate processes or substantially coincides with the radiation field in the core.

Figure 5 illustrates the effect of changing Reynolds number on the temperature field, when PL

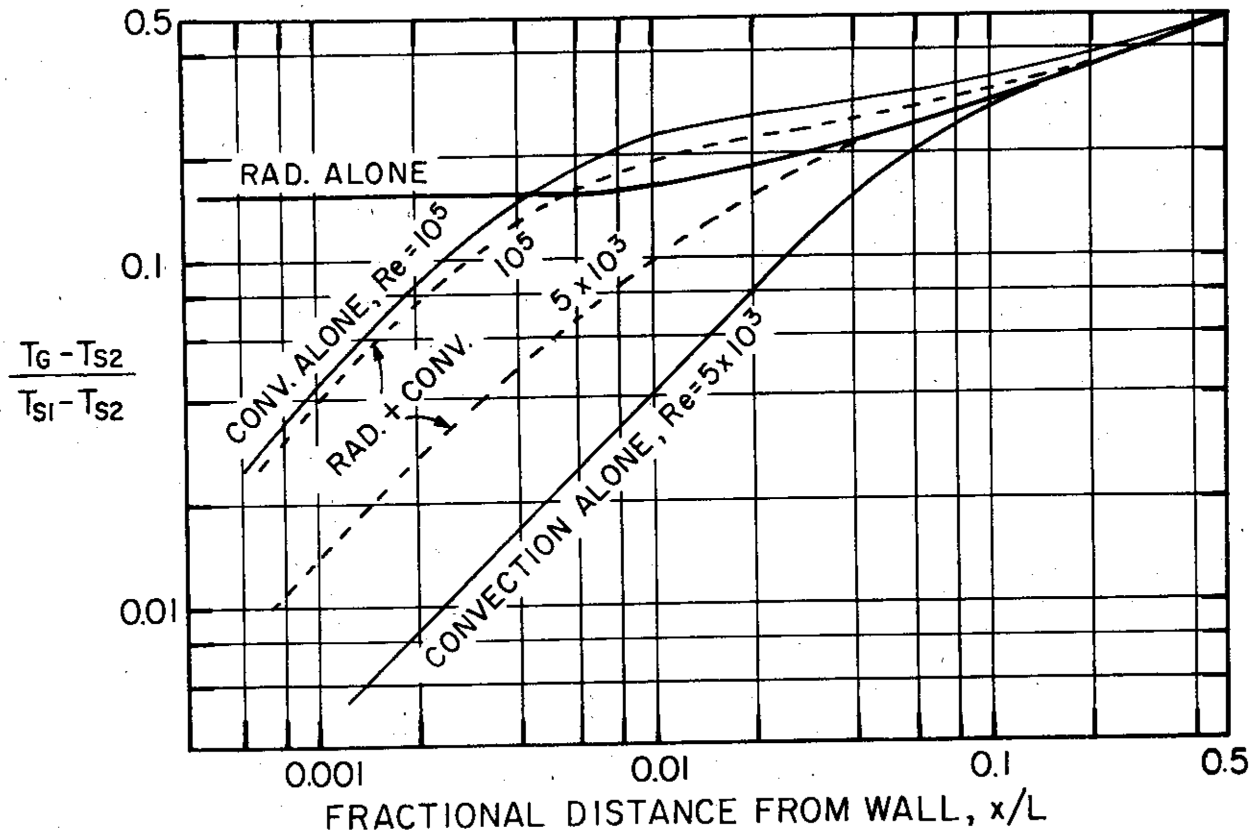

FIG. 5 – EFFECT OF REYNOLDS NUMBER ON THE TEMPERATURE PROFILE.
$PL = 0.5$ ft atm; Mean temperature = 2500R.

is held constant at 0.5 ft atm. The pure convection field is given for $Re = 5000$ and 100,000 and the pure radiation field is also shown. The field due to combined action of radiation and convection shows an interesting characteristic. At the higher Reynolds number the temperature lies between its values for pure radiation and pure convection in the core and part way into the boundary layer, out to an x/L of about 0.006; but near the wall the temperature under the influence of both mechanisms is lower than for radiation or convection alone. Radiation has been increased by the high Reynolds number flattening the core profile and thereby bringing hot gas nearer the wall where the latter can "see" it better, and convection has dropped; and the latter means a lower $\lambda dT/dx$ at the wall and a consequent downward displacement of the curve on the log plot at its left end.

In this system the assumption of a small temperature difference across the walls prevents a shift in the temperature field, anchored at its middle, from affecting viscous or momentum forces or, in consequence, the flow pattern. But the change in temperature field does of course modify both the radiation and the convection, in directions which partly compensate. Convection forces the gas temperature adjoining the wall to equal the wall temperature and thereby tends to reduce the radiation, the reduction being less the higher the Reynolds number, and finally changing sign. Radiation in modifying the temperature profile near the wall tends to increase the convection, the increase being less the higher the Reynolds number, and finally changing sign. The total flux is less than the sum of the pure convection and pure radiation flux when PL and T are high, greater when PL and T are low; but the difference is small in the present example.

The total flux has been plotted in Fig. 6, using the groups that appear in (10) as coordinates and parameters, except that the last group in the parenthesis is modified to its dimensional equivalent $3\sigma\epsilon_G T_G^3/\lambda P$, (atm ft)$^{-1}$ by inserting only the P from K_1 $(\equiv k_1 P)$. The walls are assumed black. The calculated data points, not shown, lie on the solid-line portions of the curves with a maximum deviation indicated by the cross-hatched areas at the extreme right. This indicates that the correlation groups used, (10), have adequately allowed for the variation in mean temperature from 1500 to 3500 R and the corresponding variation in emissivity. The dotted lines extending to the right indicate the approach to radiation-dominated flux which, at high enough value of the abscissa $3\sigma\epsilon_G T_G^3/\lambda P$, corresponds to constancy of the ordinate $(q/A)_{total}/(3/4)\epsilon_G(E_{GM} - E_{s_1})$ at a

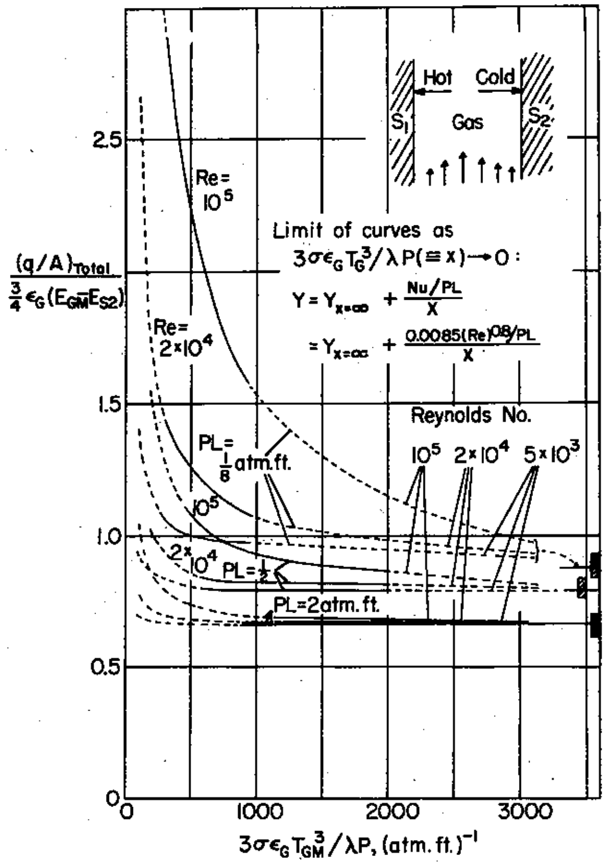

FIG. 6 – INTERACTION BETWEEN CONVECTION AND RADIATION IN FLOW BETWEEN HOT AND COLD PLATES. TOTAL FLUX FROM GAS TO COLD WALL.

Influence of Re, Pl, and $\sigma\epsilon_G T^3/P\lambda$.

value indicated by the solid lines through the centers of the cross-hatched areas. The solid lines may be extended to the left to lower values of $3\sigma\epsilon_G T_G^3/\lambda P$ by assuming that radiation and convection act independently in the convection-dominated region. On that assumption (11) is applicable, written in the form

$$\frac{(q/A)_T}{\frac{3}{4}\epsilon_G(E_{GM}-E_{s_1})} = \left(\frac{(q/A)_r}{\frac{3}{4}\epsilon_G(E_{GM}-E_{s_1})}\right)_{x=\infty} + \frac{(Nu)_c}{2PL}\,\frac{1}{\left(\frac{3\sigma\epsilon_G T_{av}^3}{\lambda P}\right)}, \tag{12}$$

where the subscript indicates the value of the bracket as $3\sigma\epsilon_G T_{av}^3/P\lambda$ approaches infinity. This equation has been plotted as the dotted lines on the left of the diagram. The left ends of the solid lines, calculated with allowance for interaction, are seen to fair gently into the dotted lines, indicating that interaction had there become unimportant.

Figure 6 illustrates well the shift from domination by Reynolds number at low values of the group $3\sigma\epsilon_G T_G^3/\lambda P$ to domination by radiation at high values, with the transition occurring sooner the higher the value of PL.

One regime on Fig. 6 is missing, the high-PL regime below the curves where, in the limit, radiation becomes a diffusion process. That regime will now be considered. Visualize a temperature field, between the plates, associated with a particular Reynolds number and Prandtl number. If the mean free path for radiation is small compared to the thickness of the laminar sublayer, i.e., if $KL \ggg 1$, where L is the wall spacing, and if the temperature variation from wall to wall is small enough to make T^3 nearly constant, then the gas acts in all respects as though it has a total conductivity λ_T equal to the sum of conductive and radiative contributions.

$$\lambda_T = \lambda_c + \frac{16\sigma T^3}{3K}$$

In consequence the temperature field is that which corresponds to $(Pr)_T = c\mu/\lambda_T$, and the relation

$$\frac{h_T}{(\lambda_T/2L)} = (Nu)_T = 0.019\left(\frac{c\mu}{\lambda_T}\right)^{1/3}(Re)^{0.8}$$

constitutes a true description of combined radiative and convective flux. The only difficulty is that the limit of applicability of this relation, – to problems in which the laminar sublayer is of the minimum order of 5 mean free paths thick, – puts it in the class of interesting but not generally useful relations.

What follows is some incomplete thinking on whether the above limitation can be eased. If the modification by the wall of the otherwise straight-line temperature relation through the conducting layer is too large to be ignored but extends only partway (distance x_1 Fig. 7) into the conduction layer, (distance x_2) it should be possible to use the last relation above for determining h_T provided its use is coupled with use of an artificial overall temperature difference, gas to wall, represented by

$$T_{gas} - T_{s_{1,a}} \equiv T_{gas} - T_{s_1} - \Delta$$

$$= (T_{gas} - T_{s_1})\left(1 - \frac{\Delta}{T_{gas} - T_{s_1}}\right)$$

where $T_{s_{1,a}}$ is the artificial or corrected wall temperature.

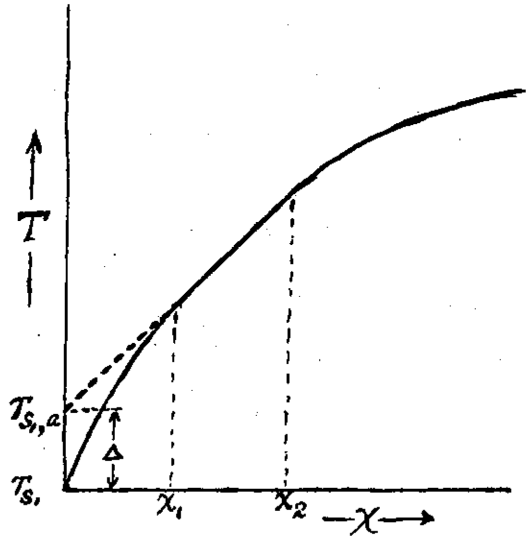

FIG. 7 – TEMPERATURE PROFILE AND PSEUDO-TEMPERATURE PROFILE IN THE BOUNDARY LAYER, WHEN ENERGY TRANSPORT IS JOINTLY BY CONVECTION AND RADIATION.

The problem is solved if Δ can be determined. It appears that Δ should conform to the relation

$$\frac{\Delta}{T_G - T_{s_1}} = f_1\left(\frac{4\sigma T^3}{\lambda K}\text{, slope at wall} = f_2\,(Pr, Re)\right)$$

The temperature profile near the wall under conditions of combined radiation and conduction, with the mean-free path lying well within the conduction layer, has been determined by Van der Held [41]; from his work the value of Δ can be estimated. The results would not be specific to the parallel plate problem chosen for discussion. Rather, the limit of validity of using λ_{c+r} in convection-type equations applicable to *any* geometry would be greatly extended.

4. OTHER AREAS OF RESEARCH ON RADIATIVE TRANSPORT

Space has prevented treatment, in any detail, of many areas in which research on radiative transport is needed, but a few of these will be considered briefly.

Isothermal Gas Masses. View-factors or exchange areas are slowly becoming available for almost any configuration of interest. Cartesian-coordinate exchange-areas among cubes and squares [23] permits build-up of any exchange factor desired. Cylindrical-coordinate exchange areas among surface rings and gas ring zones are available [25] for systems without reentry surfaces. This incompleteness should be remedied, though it presents an enormous numerical problem. Interchange areas with multiple reflection have recently become available for the wall elements of an infinite cylinder containing no gas [40]. Mean beam lengths are available for most shapes [12, 31, 32], but a few shapes perhaps require further study. In particular, 2-dimensional exchange areas are needed for gas-surface and gas-gas systems. Most important in the area of isothermal gases is the continuing accumulation of adequate descriptions of the radiating characteristics of gases and surfaces of all kinds, and the presentation of results in forms which are ready for the engineer to use. The work of Penner [24] is outstanding in this area.

Gas-wall exchange, in Geometrically Simple Systems, with Gradients in the Gas. Several relatively simple geometries have been studied, a large number remain. In addition to Adrianov and Shorin's work (l.c.), Takeuchi has studied plug-flow of real gases cooling during flow in cylinders, with allowance for axial gradients [38]; and Sarofim has allowed for radial gradients, axial gradients and recirculation in the flow of real gas through a cylinder fed at a point on its axis [33]. Usiskin and Sparrow [27] have calculated the temperature field in a uniformly distributed gray gaseous heat source confined between cold walls. Simpson [36, 37] has evaluated the radiative flux to the interior of a diathermanous droplet uniformly irradiated from without, including allowance for refractive index, conduction, thermal transients, and surface vaporization. Hoffman [39] has made somewhat similar studies on droplets. Sparrow, Usiskin and Hubbard [28] have evaluated radiative flux in gray gas confined between concentric black spheres, covering the cases of cold walls bounding a gaseous distributed heat source and inert gas between hot and cold walls.

A class of problems in this area needing attention and beginning to receive some is the evaluation of the radiative flux field in the vicinity of point and line sources forming plumes or wedges of gas being diluted and cooled by mixing with its surroundings. Both driven jets and natural-convection jets merit study.

Furnaces. Industrial furnaces continue to be designed for new chemical processing operations, and methods are needed for predicting the effects of design variables on the flux distribution to the heat-receiving surfaces. This is in many ways the most sophisticated of the problems of radiation, combining geometrical irregularity, difficulty predictable combustion patterns, and flow irregularities which have so far largely defied rigorous analysis. The best hope appears to be to use simpler systems, such as axially fed cylinders, for rigorous calculation of the effects of operating variables and, particularly, for determination of what simplifications are permissible in the model without too great a loss of accuracy in predicting performance. Progress along these lines has been reported recently [32]. Because the only method of approach to problems of this degree of complexity appears to be the zoning method and because that method can have important application in the solution of many problems of interaction, it will be reviewed briefly in the next section.

5. THE MIXED-GRAY-GAS ZONE METHOD

The two basic equations of radiative transport in an absorbing medium are an expression of the hemispherical flux through a unit area in terms of directional intensity I, and an expression representing how I varies with distance. The first is

$$q_\lambda^{\pm} = \int_{\omega = 2\pi\pm} [I_\lambda(dir)] \cos\theta \, d\omega \qquad (13)$$

where $q_\lambda^{\pm}$ represents the monochromatic hemispherical flux, in the + or − direction, per unit area; $I_\lambda(dir)$ the monochromatic intensity, or energy per unit solid angle per unit of area normal to the beam, with *(dir)* to indicate it is in general a function of its direction; θ the angle between the beam and the normal to the surface on which $q^{\pm}$ is based; ω the solid angle; and $2\pi \pm$ indicates integration over 2π steradians on the side of the emergent flux, + for q^+, − for q^-. The second relation, omitting allowance for scatter, is

$$-\frac{dI_\lambda}{dr} = K_\lambda I_\lambda - K I_{B,\lambda}(T) \qquad (14)$$

where r is distance along the beam; K_λ is the absorption coefficient or reciprocal mean free path; and $I_{B,\lambda}$ is the intensity of black-body radiation at the local temperature T.

Solution of these equations is very difficult except for the case of unidirectional flux where the loci of constant values of I and I_B are planes of constant $r\cos\theta$ and where $d\omega$ is in consequence expressible in $\theta(d\omega = 2\pi\, d(\cos\theta))$; and even for that simplest of geometries most authors have restricted their treatment to "gray" gases. For the many problems of the engineer not falling into the few cases manageable with the above equations, – problems of odd furnace shapes, of temperature fields influenced by flow, of simultaneous axial and radial temperature gradients, of jet mixing, – there appears to be no alternative to zoning the system. One thereby replaces the integro-differential equation plus complex boundary conditions by a system of as many simultaneous linear equations governing radiation as there are gray surface zones and scattering gas zones, solves for a set of intermediate coefficients which are then fed into a system of as many simultaneous and generally non-linear total-energy balances as there are unknown zone temperatures. A description of the method, without derivations, follows:

1. Describe the gas total emissivity and absorptivity empirically as though the gas were a mixture of a few gray gases, in the manner described under restriction 2 in the section above on Interaction.

$$\epsilon_G = \sum_1^n a_n'(T_G)(1 - e^{-K_n L}) \qquad \sum_0^n a_n = \sum_0^n a_n' = 1$$

$$\alpha_{G,S} = \sum_1^n a_n(T_s)(1 - e^{-K_n L})$$

Experience indicates n need not exceed 3; 2 often suffices.

2. Zone the system into the coarsest zoning consistent with the accuracy desired, in a manner dictated by the geometry of the problem and by prior knowledge, or guess, of where steep gradients in the temperature field necessitate fine-scale zoning.

3. Evaluate direct-interchange areas $\overline{ss}$, $\overline{sg}$, $\overline{gg}$, for each of the n values of KB.

4. Set up and solve system of simultaneous linear equations representing radiation balances on all zones, using interchange areas based on absorption coefficient K_1.

Surface zone i:

$$\sum_j \left(\overline{s_j s_i} - \delta_{ij}\frac{A_i}{\rho_i}\right) W_j + \sum_j \overline{g_j s_i}\, W_j' = -\frac{A_i \epsilon_i}{\rho_i} E_i$$

Volume zone i:

$$\sum_j \overline{s_j g_i} W_j + \sum_j \left(\overline{g_j g_i} - \delta_{ij}\frac{4KV_i}{b}\right) W_j' = 4KV_i \frac{1-b}{b} E_i'$$

Solve for each ${}_jW_i$ and ${}_jW_i'$ (Definitions of ${}_jW_i$ and ${}_jW_i'$ appear in nomenclature table, also δ_{ij}).

5. Evaluate total-interchange areas.

$$\overline{S_i S_j} = \frac{A_i \epsilon_i}{\rho_i}({}_jW_i - \delta_{ij}\epsilon_i)$$

$$\overline{G_i G_j} = 4KV_i \frac{1-b}{b}({}_jW_j' - \delta_{ij}(1-b))$$

$$\overline{S_i G_j} = \frac{A_i i}{\rho_i}\, {}_jW_i = 4KV_j \frac{1-b}{b}({}_iW_j)$$

6. Repeat steps 4 and 5 n times, once with each value of K needed to describe the gas.

7. Estimate the temperature field, and for it evaluate

$a_n(T_i)$ for each surface zone i and each n, and

$a_n'(T_i)$ for each gas zone i and each n.

8. Evaluate the directed total-interchange areas

$$\overrightarrow{S_iS_j} = \sum_0^n [a_n(T_i)](\overline{S_iS_j})_n \;;\; \overleftarrow{S_iS_j} = \sum_0^n [a_n(T_j)](\overline{S_iS_j})_n$$

$$\overrightarrow{S_iG_j} = \sum_1^n [a_n(T_i)](\overline{S_iG_j})_n \;;\; \overleftarrow{S_iG_j} = \sum_1^n [a_n(T_j)](\overline{S_iG_j})_n$$

$$\overrightarrow{G_iG_j} = \sum_1^n [a_n(T_i)](\overline{G_iG_j})_n \;;\; \overleftarrow{G_iG_j} = \sum_1^n [a_n(T_j)](\overline{G_iG_j})_n$$

9. Formulate the total energy balances on those zones of unknown temperature.

Balance on A_i:

$$\sum_j \overrightarrow{S_jS_i} E_j + \sum_j \overrightarrow{G_jS_i} E_j' - A_i \epsilon_i E_i' + \left\{\begin{array}{l}\text{(Reception or generation of energy by}\\ A_i \text{ due to non-radiative mechanisms}\\ \text{dependent on } T_i \text{ and on those } T_j\text{'s con-}\\ \text{tiguous to } T_i) - \\ \text{(storage rate or withdrawal rate)}\end{array}\right\} = 0$$

This gives a system of simultaneous equations, generally non-linear, of total number unrelated to the number encountered in step 4. If the temperature field differs from that assumed in step 7, repeat from 7 on. Note that steps through 6 produce a complete description of the radiating characteristics of the system. Studies of the effects of changes in flow rate or flow pattern, feed stream preheat, temperature distribution of the controlled part of the walls, or any factor not affecting the system shape or size, wall emissivity, or gas composition, — all these studies are made by changes from step 7 on. But the chore of going from step 1 through 6 indicates the need for restraint in choosing the number of zones and the n for the gas!

The derivation of the above steps, together with the discussion of their validity, has been presented elsewhere [29]. A formulation of energy transfer from gases with temperature gradients in terms of the basic temperature — dependent parameters governing gas emission has been given by Penner [22]. In comparison with the method summarized here, the latter formulation is more rigorous in its allowance for property variations along the direct radiation path, but less general in that it does include radiative transport from source to sink via indirect paths involving wall reflection or gas scatter. Moreover, the complicated form of the equations, the difficulty of applying them to irregular shaped enclosures such as furnaces, and the uncertainty as to the correct values of the parameters describing the fine structure of the spectra of molecules of interest probably make unjustifiable the use, in most engineering calculations, of a more rigorous description of the effects of non-grayness of gases and their temperature response than that outlined above.

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