Heat Transfer by Natural Convection

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LECTURE

1. INTRODUCTION

Heat transfer, starting as a branch of engineering thermodynamics at the end of the 19th century has in the last 50 years changed to a science of its own being the topic of this international conference.

As I was given the honor to read here one of the four general lectures of reviewing character, I chose the theme "heat transfer by natural convection" mainly because years ago I made some contributions to this field and I have always followed with interest its further development.

Today heat transfer at high velocities up to the re-entry speed of satellites is in the foreground of interest. Here extremely intensive heat flows occur because all the kinetic energy of the vehicle transforms into heat, which has to be dealt with because the content of the satellite and its construction only withstands moderate temperatures.

Our industrial heat transfer apparata usually operate with rather high flow velocity at so-called forced convection because the heat transfer for a given size of an apparatus increases with higher velocities of the fluids, liquids or gases washing the heat exchanging surfaces.

If the velocity of the fluid is only the result of the buoyancy of the heated fluid in the field of gravity its kinetic energy is produced thermodynamically, no pumping energy is necessary and we speak of natural or free convection.

Heat transfer by natural convection is of great practical interest: most devices for heating our houses deliver heat mainly by natural convection. The heat transfer through layers of gases and liquids is usually increased by free convection. But these velocities are rather small and need delicate instruments for being measured. With larger dimensions the velocities increase substantially as we see in the atmosphere where the differences of air temperature produce winds and storms.

However, also at dimensions realizable in a laboratory great velocities of free convection are attainable if the fluid nears its critical state, because here the coefficient of thermal expansion which produces buoyancy goes towards infinity. On the other hand the specific heat of the fluid which determines the amount of heat transport per unit mass of the fluid also nears infinity. In this way a vertical tube filled with the critical quantity of a fluid, heated below and cooled above transports at the critical temperature of the fluid only by natural convection the same amount of heat as a solid rod of the dimensions of the tube having a heat conductivity of some thousand times that of copper.

The velocity of natural convection can also be increased very substantially in rotating bodies, for instance in the drums and blades of turbines where the centrifugal acceleration easily reaches values of 30,000 times that of gravity.

2. GENERAL THEORY

For the complete solution of a problem of heat transfer we need the field of temperature and the field of velocity in the neighbourhood of the heat transmitting solid surface. Both fields are determined by the differential equations of heat conduction in a moving fluid and by the differential equations of the flow of a viscous liquid with nonuniform temperature together with the laws of preservation of mass and of energy, and the solution has to satisfy the boundary conditions of the problem.

At forced convection usually it is allowable to assume that the forces of buoyancy are small enough not to influence the flow. Then the field of velocity can be calculated without knowing the field of temperature whose determination in this way is facilitated.

However, in the general case, and if free convection prevails, both fields are linked with each other in a complicated system of non linear differential equations.

Therefore, complete mathematical solutions are only possible for a small number of simple problems, and for most practical purposes we have to rely on experiments.

In two famous papers published in 1910 [1] and 1915 [2] W. Nusselt showed that by introducing dimensionless variables the amount of experimental work can be reduced substantially, because the result of an experiment does not depend upon all possible variables but only upon the values of a smaller number of dimensionless groups combining sets of variables in the form of power products. In this similarity theory of heat transfer Nusselt replaces the surface heat transfer coefficient h already introduced in 1701 by Isaac Newton in his equation

$$q = h \cdot A \cdot \theta \tag{1}$$

(q is the heat rate, A the surface area and θ the difference of temperature) by the so-called Nusselt-number

$$Nu = \frac{hl}{\lambda} \tag{2}$$

where l is a characteristic length of the heat releasing body in question and λ the thermal conductivity of the fluid. Nu depends upon the coordinates at the surface of the body and is proportional to the gradient of temperature in the fluid layer touching it.

Another dimensionless group essential in heat transfer as well as in fluid mechanics is the Reynolds number

$$Re = \frac{wl}{\nu} \tag{3}$$

where w is a flow velocity usually given by the boundary conditions and ν the kinematic viscosity of the fluid. A third dimensionless group is the Prandtl number

$$Pr = \frac{\nu}{a} \tag{4}$$

where $a = \frac{\lambda}{c_p \rho}$ is the thermal diffusivity of the fluid. For air, steam and other gases Pr is not far from unity, for nonmetallic liquids it is greater than unity up to some thousands for lubrication oils. For liquid metals it has very low values down to 0.01 for liquid sodium.

Finally the dimensionless group governing free convection problems is the Grashof-number

$$Gr = \frac{g \beta l^3 \theta}{\nu^2} \tag{5}$$

where g means the acceleration of gravity or by centrifugal forces and β the coefficient of thermal volumetric expansion.

Instead of Grashof number sometimes the Rayleigh number

$$Ra = Gr \cdot Pr = \frac{g \beta l^3 \theta}{a \nu} \tag{6}$$

is used. The theory of similarity shows that heat transfer with free convection under steady state condition can be expressed by an equation of the form

$$Nu = f(Re, Gr, Pr, u, v) \tag{7}$$

where u and v are coordinates on the heat transmitting surface. In practical applications usually only the mean overall Nusselt number is of interest, to be found by integration which reduces the variable coordinates to the known dimensions of the surface.

If there is no forced convection, the Reynolds number is to be omitted, and if the velocities of free convection are rather small as in liquids of high viscosities or with small dimensions of the fluid, the dependency upon Gr and Pr reduces to a function only of its product $Ra = Gr \cdot Pr$ also called Rayleigh number and can be written

$$Nu = f(Ra) \tag{8}$$

This simplification can be allowed for small values of Ra, when the forces acting upon a fluid volume by viscosity, depending upon the gradient of the velocity itself, are great compared with the forces of mass acceleration which are proportional to the square of the velocities.

If the steady state is not yet attained, the transfer of heat also depends upon time t which enters the general solution in the form of another non dimensional group

$$Fo = \frac{at}{l^2} \tag{9}$$

called Fourier number.

3. TYPICAL PROBLEMS OF FREE CONVECTION

Because a short lecture cannot cover the whole field of natural convection I now shall try to show the development of our knowledge of some typical problems in this field.

a. The Vertical Flat Plate. The vertical flat plate is the simplest form of a heating device, for instance as the surface of a stove or the radiator of a central heating installation. Measurements of the field of temperature before a vertical hot plate with

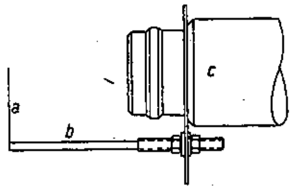

FIG. 1 – QUARTZ FIBRE ANEMOMETER BY E. SCHMIDT
a. Quartz fibre
b. Support pin
c. Tube of the microscope

very thin thermocouples were first made by W. Nusselt and H. Jürges [3], but they could not measure the field of velocity in default of a suitable instrument for these small velocities. E. Schmidt solved this problem with his quartz fibre anemometer shown in Fig. 1 consisting of a straight fibre of quartz only about 0.02 mm thick, 1 to 2 cm long, fixed at one end, and placed in the field of velocity. This quartz fibre is bent elastically by the flow of air passing it perpendicularly to its length. The deflection of the end of the fibre is measured with the help of a microscope, having several centimeters' distance from the fibre to the objective lense, in order not to disturb the flow where it is measured. This instrument allowing the measurement of air velocities down to a few millimeters per second, and a thin thermocouple 0.02 mm in diameter enabled E. Schmidt and W. Beckmann [4] to measure the whole fields of velocity and temperature before a vertical hot plate. Curves of temperature t and vertical velocity u in horizontal planes in different heights x above the lower edge of the plate are given in Fig. 2.

The experiments proved that the hot air in motion before a flat plate is confined to a rather thin layer which allows the simplification of boundary layer theory. Having these experimental results before him, E. Pohlhausen [4] found the mathematical solution of the problem. By introducing instead of the vertical coordinate x and the horizontal coordinate y the independent variable $cy/\sqrt[4]{x}$ where $c = 5{,}885$ is a parameter given by the viscosity and buoyancy of the air at plate temperature t_0, the partial differential equations of velocity and temperature change to nonlinear but ordinary ones containing only derivations up to third order to the new independent variable. If the nondimensional air temperature t/t_0 and instead of the velocity value u the value $u/26{,}88\sqrt{x}$ are plotted against the independent variable $5{,}885\ y/\sqrt[4]{x}$ as done in Fig. 3,

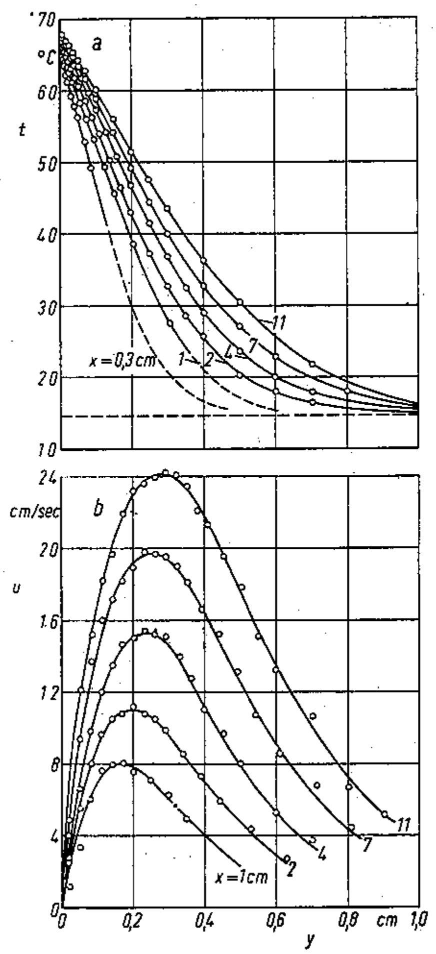

FIG. 2 – HORIZONTAL CROSS SECTION OF (a) THE FIELD OF TEMPERATURE t AND (b) THE VERTICAL COMPONENT μ OF VELOCITY BEFORE A VERTICAL PLATE 12 cm HIGH AND HEATED TO ABOUT 66 C IN AIR OF 15 C, MEASURED AT DIFFERENT VERTICAL DISTANCES x ABOVE THE LOWER EDGE OF THE PLATE

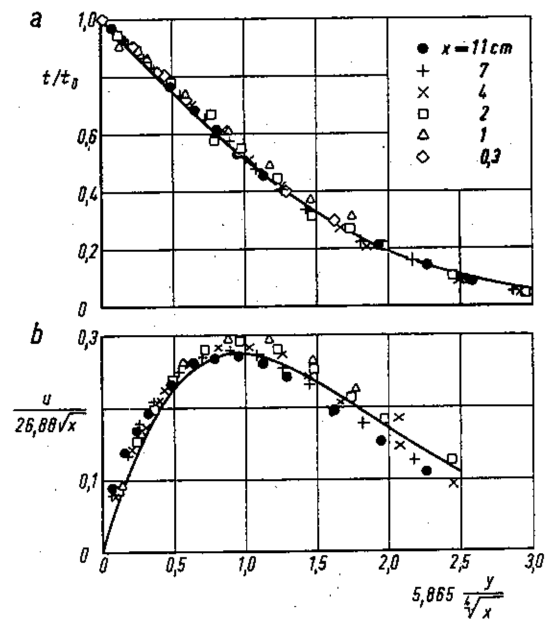

FIG. 3 – THE MEASURED POINTS OF FIG. 2 ARE PLOTTED HERE AS NON DIMENSIONAL QUANTITIES t/t_0 AND $\frac{u}{26{,}88\sqrt{x}}$ ABOVE THE ABSCISSA $5{,}885\frac{y}{\sqrt[4]{x}}$. IN THIS WAY THE CURVES FOR DIFFERENT HEIGHTS x ABOVE THE LOWER EDGE OF THE HEATED PLATE COINCIDE RATHER WELL IN FIG. 3a AND FIG. 3b WITH THE CURVES OF TEMPERATURE AND VELOCITY FOUND BY CALCULATION.

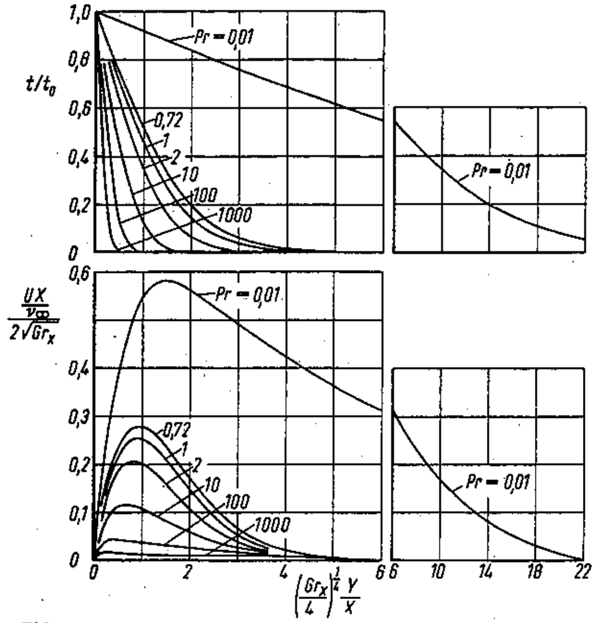

FIG. 4 – CURVES OF NONDIMENSIONAL TEMPERATURE t/t_0 AND OF A NONDIMENSIONAL VELOCITY FUNCTION PLOTTED AGAINST AN ABSCISSA PROPORTIONAL TO $\frac{y}{\sqrt[4]{x}}$

a and b, we see that all observed points join in the two full curves found by numerical integration of the differential equations.

These calculations were made for air with a Prandtl number of 0.733. Later Schuh [5] gave results for some Prandtl numbers > 1 and finally Ostrach [6] covered the whole range $0.01 < Pr < 1000$. His results in dimensionless coordinates are given by the curves of Fig. 4, which show that for large Prandtl numbers the field of velocity extends much further into the fluid than the field of temperature.

Ostrach summarized his calculations of the laminar boundary layer in the equation for the mean Nusselt number:

$$Nu = f(Pr)(Ra)^{\frac{1}{4}} \tag{10}$$

where the function $f(Pr)$ is given for some values of Pr in the following table

Pr =	0,01	0,733	1	10	100	1000
$f(Pr)$ =	0,242	0,518	0,535	0,619	0,652	0,665

For most practical purposes the full knowledge of the fields of temperature and velocity is not necessary. Usually it is sufficient to know the local heat transfer coefficients and to have an idea up to what distance from the hot surface the increase of temperature of the fluid extends. For plane and cylindrical bodies this knowledge can be got optically with the help of the following simple installation originated by E. Schmidt [7]. In a long corridor with quiet air of constant temperature the heated body, for instance a plate or a cylinder with horizontal axis, is suspended. At a distance of about 50 m from the heated body a small intensive source of light, for instance an

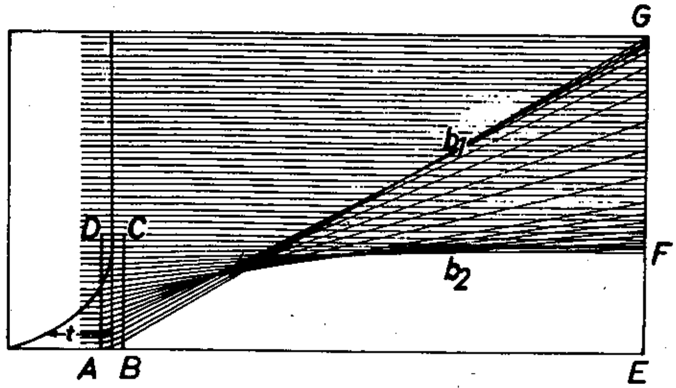

FIG. 5 – THE WAY OF LIGHT RAYS DEFLECTED IN THE BOUNDARY LAYER OF A HEATED SURFACE

electric arc lamp, is placed in such a way that the practically parallel rays of light just touch the surface of the body, and after another way of 20 to 30 m fall on a white screen or a photographic plate. In this way the layer of warm air surrounding the heated body and having a gradient of temperature and density imports to the rays of light a small curvature proportional to the gradient of density. The displacement of the light rays on their short way of a foot or less along the heated body attain only a few tenths of a millimeter but after a way of another 100 feet it reaches the order of magnitude of centimeters. In Fig. 5 the way of the parallel rays of light passing the layer ABCD of warm air with a temperature given by the curve t as a function of the distance from the heated plate AB is shown. The thickness AD of the layer of warm air is given in a highly increased scale, in reality AD is much smaller than the length AB of the heated surface. As to be seen in the figure the parallel rays of light coming from the left are deflected and after some run they fill the area between the two focal curves b_1 and b_2 until they fall on the screen EG and produce the picture.

Typical photographs obtained in this way are given in the following figures. Fig. 6 shows the heat

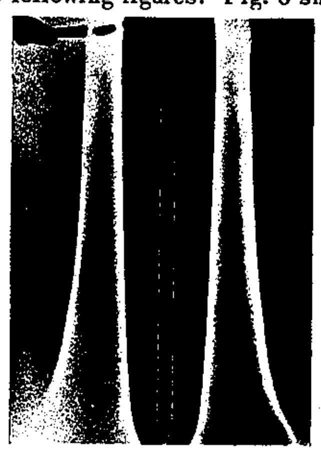

FIG. 6 – SCHLIERENPHOTOGRAPH OF THE FIELD OF TEMPERATURE IN FRONT OF A PLANE VERTICAL PLATE 12 cm HIGH AT A TEMPERATURE OF ABOUT 70 C ABOVE THAT OF THE SURROUNDING. THE DOTTED LINE IS THE CONTOUR OF THE SHADOW OF THE PLATE IF NOT HEATED.

FIG. 7 – SCHLIERENPHOTOGRAPH OF THE FIELD OF TEMPERATURE SURROUNDING A HORIZONTAL CYLINDER OF 5 cm DIAMETER. THE DOTTED CIRCLE IS AGAIN THE SHADOW CONTOUR OF THE NON HEATED CYLINDER

transfer at a vertical plate. If not heated its shadow contour is given by the dotted lines. At a temperature of about 70 C above that of the surrounding air, the plate is enclosed by a deep black shadow, expanding as far as the temperature of the air is increased. The outer bright lines converging from below to above correspond in their distance from the surface of the plate to the heat transfer coefficient or Nusselt number decreasing from below to above. A

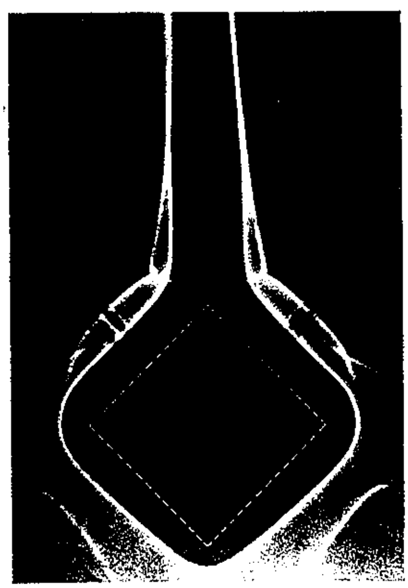

FIG. 8 – SCHLIERENPHOTOGRAPH OF A HORIZONTAL CYLINDER WITH SQUARE CROSS SECTION

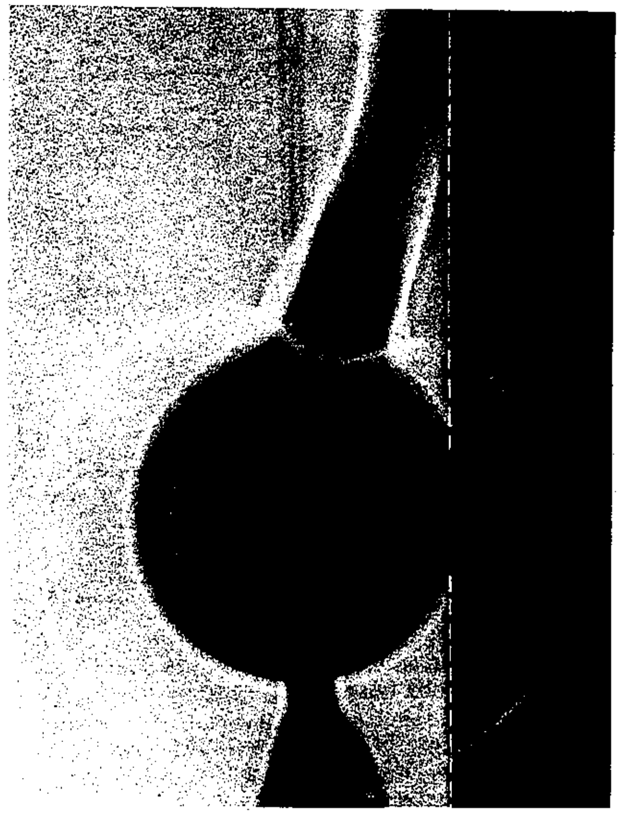

FIG. 9 – SCHLIERENPHOTOGRAPH OF A HEATED HORIZONTAL CYLINDER NEAR A VERTICAL NOT HEATED PLATE. THE DOTTED LINES DENOTE AGAIN THE CONTOUR OF BOTH BODIES

similar photograph taken at a horizontal cylinder of 5 cm diameter and 30 cm long is shown in Fig. 7. Here the heat transfer coefficient is given by the distance of the heart-shaped outer border of the bright area from the dotted contour of the shadow of the non heated cylinder. We learn that the heat transfer coefficient has a flat maximum at the deepest part of the cylinder and decreases around it to a pronounced minimum above the cylinder, produced by the convergence of the two flows of hot air ascending around the cylinder and leaving it in vertical direction as shown by the black area. The corresponding photograph of a square cylinder of Fig. 8 is understood without explanation.

The possibilities of Schmidt's method are shown for instance in Fig. 9, where a flat vertical plate indicated by the dotted straight line is brought closer to the cylinder of Fig. 7. We see that the heart-shaped curve gets a swelling where the flat plate nears the cylinder, indicating that here the heat transfer coefficient locally increases. In the upper part of the picture the ascending stream of warm air approaches the plate, which at this side prevents the access of the surrounding air.

The experimental installation for getting these schlieren photographs is very simple and cheap, because only an electric arc lamp is needed and also large objects can easily be photographed in this way.

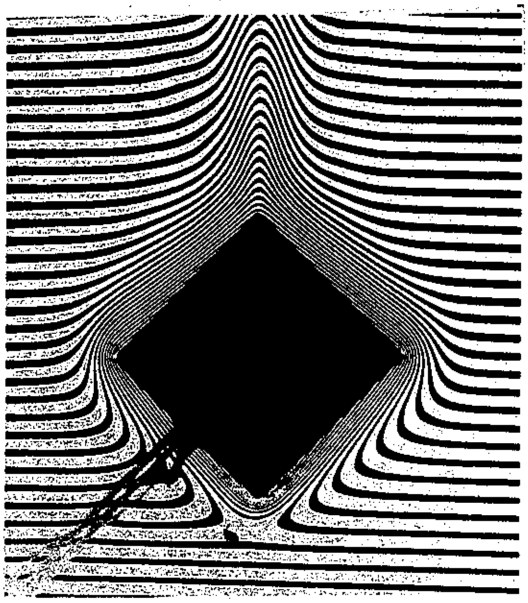

FIG. 10 – INTERFEROMETRIC PHOTOGRAPH OF A HORIZONTAL CYLINDER OF 1 cm² CROSS-SECTION (by courtesy of H. Schardin)

Similar results may be obtained with the help of an interferometer following Mach-Zehnder, but this is an expensive device, and confined in its field of observation by the size of optical lenses and mirrors. An interferometric photograph of a square cylinder of 1 cm^2 cross-sectional area, taken by Schardin [8] is shown in Fig. 10. In such interferograms the local heat transfer coefficient is proportional to the reciprocal distance of the interferometric fringes at the surface of the heat releasing body.

The heat transfer from a vertical plate, which may be called the classical problem of natural convection has been studied by different other authors. Saunders, who also used Schmidt's optical method [9] found in experiments with water that at some height above the lower edge of the plate, corresponding to a Rayleigh number of the order of magnitude 10^9, the laminar flow changes to turbulence. Touloukian, Hawkins and Jakob [10] measured the heat transfer from a vertical cylinder to water and ethyleneglycol and summarized their results for laminar convection ($2 \cdot 10^8 < Ra < 10^{10}$) in the equation

$$Nu = 0.726\,(Gr \cdot Pr)^{\frac{1}{4}} \qquad (11)$$

which changes for turbulent convection at higher Rayleigh numbers into

$$Nu = 0.0674\,[Gr\,(Pr)^{1.29}]^{1/3} \qquad (12)$$

For laminar motion the experiments of different authors are in rather good agreement with each other and with theory. The change to turbulence is not so well known because it needs some length as is nicely to be seen in the interferograms of Eckert and Soehngen [11] shown in Fig. 11. The three pictures are taken at the same plate at different heights. In the left and lowest picture the boundary layer is still

FIG. 11 – INTERFEROGRAMS SHOWING THE CHANGE OF THE LAMINAR NATURAL CONVECTION BEFORE A HEATED VERTICAL PLATE TO TURBULENCE

laminar, in the middle turbulent waves start and increase in the right picture.

Goldstein and Eckert [12] studied the transient development of free convection at a very thin (0.001 in.) foil of stainless steel 4 in. broad and 6.5 in. high in water with the help of a Mach-Zehnder interferometer. An interferogram of the heated foil with boundary layers is given in Fig. 12. They also found that a time of the order of magnitude of a minute is necessary to reach the steady state. During the transient period the water almost behaves like a solid body. The steady state results of these experiments are in good agreement with Sparrow and Gregg [13], who gave the mathematical solution for the laminar boundary layer with uniform surface heat flux.

Around a horizontal cylinder of 5 cm diameter the field of temperature and velocity at natural convection was measured with thermocouple and quartz fibre anemometer in 1933 by Jodlbauer [14] and in 1936 Hermann [15] gave the mathematical solution in good agreement with experiment.

However, already in 1912 Langmuir [16] had treated the heat convection of very thin cylinders as they are used as filaments in electric bulbs. He found that the area of heated air surrounding the thin wire is many times thicker than its diameter and the boundary layer theory is not applicable. Therefore, he proposed to assume a cylinder of quiet air surrounding the thin wire and Elenbas [17] developed

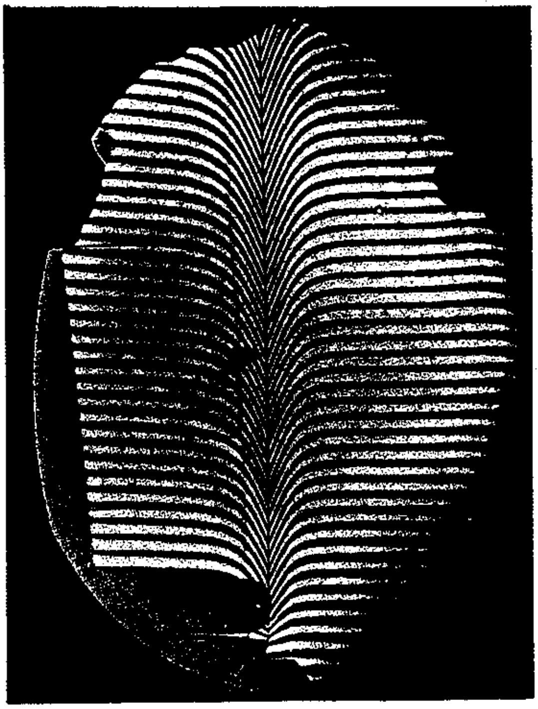

FIG. 12 – INTERFEROGRAM OF THE FIELD OF TEMPERATURE OF A HEATED STAINLESS STEEL FOIL IN WATER

a theory based on this idea in good agreement with measurements at electric bulbs.

A schlierenphotograph, taken by E. Schmidt, of a heated wire, only 0.1 mm thick, is given in Fig. 13. The wire is spanned vertically to the plane of the figure between two conductors, the shadow of which cross the picture from left to right. The wire is surrounded by a black shadow many times larger in diameter, which in a first approximation may be treated as a cylinder of motionless air.

b. Heat Transfer Through Fluid Layers. In layers of liquids or gases limited by solid walls of different

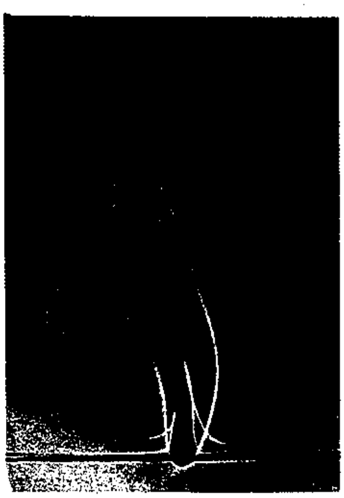

FIG. 13 – SCHLIERENPHOTOGRAPH OF A HEATED STRAIGHT WIRE ONLY 0,1 mm IN DIAMETER SPANNED VERTICALLY TO THE PLANE OF THE FIGURE

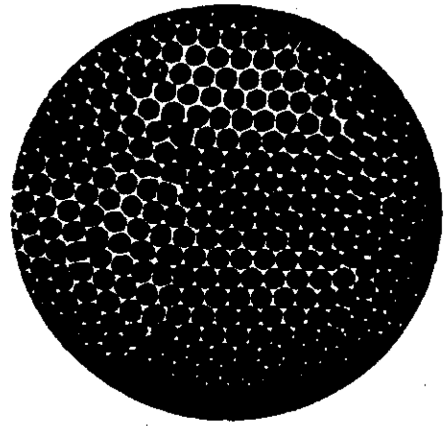

FIG. 14 – PATTERN OF NATURAL CONVECTION MOTION IN A HORIZONTAL LAYER OF PARAFFIN OIL

temperature heat is not only transported by conduction but also by radiation and by natural convection if the heat does not flow vertically from above to below. The transport by radiation can be substantially diminished by using bright metal surfaces for instance of thin aluminium foil as it is done in the so-called alfol-insulation proposed by E. Schmidt [18]. The heat transport by natural convection presupposes a certain thickness of the fluid layer or more exactly said: it starts not before a certain Rayleigh number is attained. Already in 1900 Benard [19] showed in nice photographs taken with parallel light passing a layer of liquid paraffin heated from below, that the transport of heat by natural convection produces a pattern of hexagonal cells, as to be seen in Fig. 14. The warmer fluid ascends in the center of each cell and returns around its perimeter. Lord Rayleigh [20] found in 1916 by considerations of stability that in a horizontal liquid layer heated from below and cooled above natural convection starts at a critical Rayleigh number

$$Ra = Gr \cdot Pr = 1700$$

when the thickness of the layer is introduced as characteristic length into the Grashof or Rayleigh numbers, but he did not discuss the contribution of natural convection to heat flow.

The first reliable measurement of heat transfer by natural convection in plane and in cylindrical layers of air were published in 1927 by E. Schmidt [18]. Similar experiments with air covering a broader range of conditions were made by Mull and Reiher [21] in 1930. Measurements with water in horizontal layers by R. J. Schmidt and Saunders [22] confirmed the theoretical value of the critical Rayleigh number. Recently E. Schmidt and P. L. Silveston [23, 24] made experiments with liquids of different viscosities. Their results are given in Fig. 15, where the non-dimensional ratio λ_k / λ of the apparent or equivalent heat conductivity λ_k including convection to the true heat conductivity λ of the fluid is plotted against Rayleigh number. We see that for $Ra < 1700$ there is only true conduction, but at this critical value found

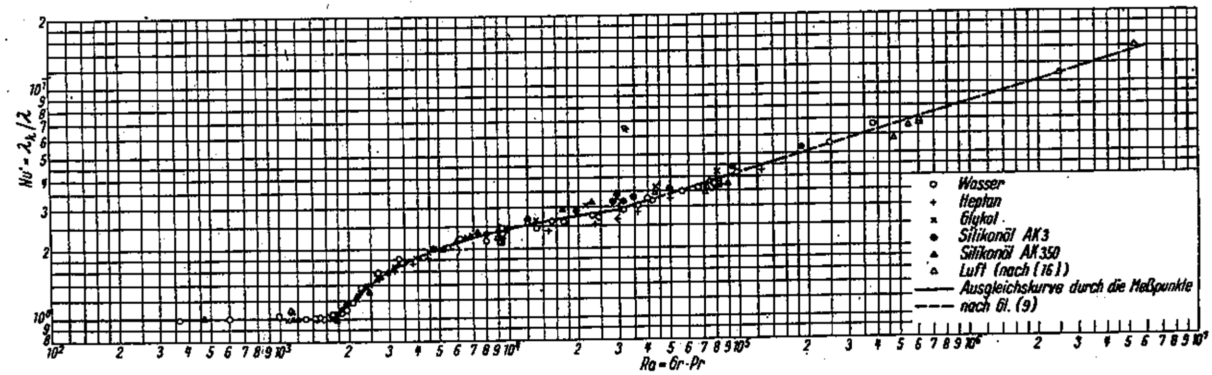

FIG. 15 – TRANSPORT OF HEAT BY NATURAL CONVECTION IN A HORIZONTAL LAYER OF LIQUID HEATED BELOW AND COOLED ABOVE, PLOTTED AS THE RATIO λ_k / λ OF APPARENT TO TRUE CONDUCTIVITY AGAINST RAYLEIGH NUMBER

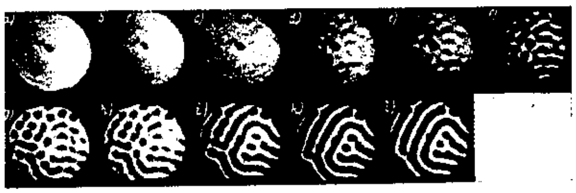

FIG. 16 – DEVELOPMENT OF THE PATTERN OF FREE CONVECTION MOTION IN A HORIZONTAL LAYER OF A LIQUID 7 mm THICK WITH INCREASING TIME AND RAYLEIGH NUMBER

Picture	*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*k*	*l*
Time since start of heating min.	0	50	52	55	56	60	65	75,5	100	120	167
Rayleigh number	0	1590	1640	1700	1750	1800	1900	2000	2450	3010	4560

theoretically by Lord Rayleigh convection begins in form of hexagonal cells which with increasing Rayleigh number change to rolls until about at $Ra = 5000$ the regular laminar flow becomes turbulent, and the pattern disappears. The development of this phenomenon is to be seen in the set of photographs in Fig. 16 taken at different times after the start of the heating. The table below the figure gives these times and the corresponding Rayleigh numbers.

In vertical layers the heat transfer does not only depend upon the thickness of the layer but also upon its height. In Fig. 17 based upon unpublished measurements with air by Kakuschky in my laboratory the ratio λ_k / λ is plotted against the ratio l/d of the height l to the thickness d of the layer, and the curves correspond to different Rayleigh numbers with d as characteristic length. The curves show that decreasing the height of a layer of constant thickness or by subdividing it by thin partitions the transfer of heat by convection increases slowly up to a maximum at a ratio l/d of about 0.25 and then falls down with decreasing height to $\lambda_k / \lambda = 1$ because in such low channels viscosity prevents any convection.

In 1949 Jakob [25] correlated all data available at that time in his book.

A thorough and systematic investigation of heat transfer and convection in enclosed plane air layers in horizontal, vertical and in oblique positions was published by de Graaf and van der Held [26]. They observed and described also interesting flow patterns. Theoretical studies of the problem were made by Batchelor [27] and by Poots [28] from which we learn that for not-too-small Rayleigh numbers and ratios l/d the warm and the cold surface have their separate boundary layers including in the middle of the cavity

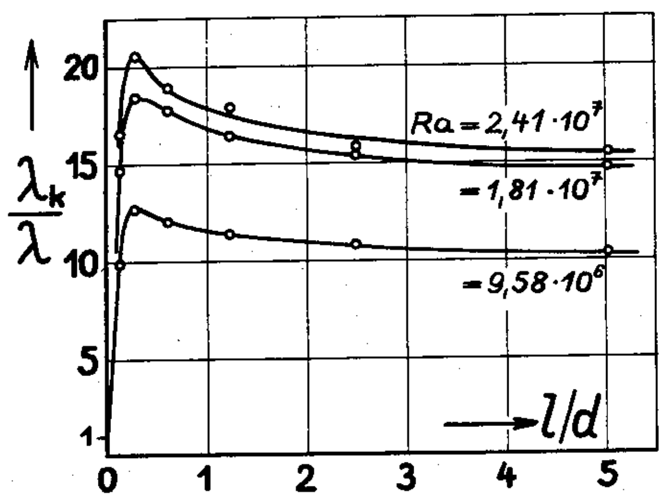

FIG. 17 – INFLUENCE OF THE RATIO l/d (HEIGHT TO THICKNESS) OF A VERTICAL LAYER OF AIR UPON ITS APPARENT OR EQUIVALENT HEAT CONDUCTIVITY λ_k MADE DIMENSIONLESS BY THE TRUE CONDUCTIVITY λ

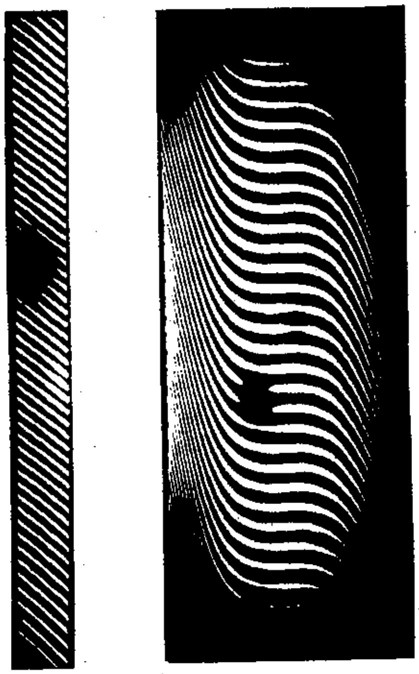

FIG. 18 – INTERFEROGRAMS OF VERTICAL LAYERS OF AIR
a) 0,75 cm thick with 5,6 C difference of temperature
b) 3,5 cm thick with 108 C difference of temperature

an almost isothermal core with constant vorticity.

These theoretical results are nicely confirmed by interferograms of Eckert and Carlson [29] given in Fig. 18. The left photograph is taken at a layer 0.75 cm thick with a temperature difference between both surfaces of only 5.6 C. The optical system is adjusted in such a way that a field of uniform temperature yields a system of horizontal interference fringes, and a constant gradient of temperature produces an even inclination of the fringes. Thus in Fig. 18a there is only pure conduction. However, in Fig. 18b taken at a layer 3.5 cm thick with a temperature difference of 108 C the fringes are considerably distorted. Along both surfaces left and right they are rather steep because here is a high gradient of temperature, but in the middle they are almost horizontal corresponding to a region of constant temperature in agreement with the theory.

c. Heat Transport at High Rayleigh Number and by Fluids near Their Critical States. As the Rayleigh number is proportional to the third power of the characteristic length, in the atmosphere with a thickness of many kilometers it reaches high values and very intensive convection is possible, as winds and storms show. But Rayleigh number also embraces the expansion coefficient β, the acceleration g, and in its denominator the viscosity ν. Now the coefficient of thermal expansion increases considerably if we near the critical state of a substance and at this very point it goes towards infinity. Therefore, under these conditions the buoyancy forces produce very intensive convection all the more as at the critical point the viscosity of all liquids has its smallest possible value. In addition the transport of heat by a given mass depending upon the heat capacity of the medium is also very much facilitated because towards the critical point the specific heat also goes towards infinity. In order to confirm these theoretical considerations E. Schmidt [30] experimented with cylindrical tubes in vertical position heated at the lower end and cooled above, because the high values of the critical pressure make it difficult to use flat layers of considerable horizontal area. (Carbon-dioxide has its critical point at a temperature of 31 C and a pressure of 75.8 atm, for ammonia the values are 132 C and 115 atm and for water 374 C and 225.6 atm.) The experimental set-up used by Schmidt is shown in Fig. 19. The tube a with a length of 2 m and an internal diameter of 40 mm is heated at its lower end by an electric resistance heater b and cooled at its upper end with water flowing through the jacket c. At its lower end the tube is connected by a capillary d to another tube, which can be held at any constant temperature, with the help of a liquid supplied from a thermostat and filling the jacket f.

Both tubes a and e are filled with such a quantity of the fluid that at the critical temperature it is near its critical pressure. The purpose of the second tube e is to press by thermal expansion more or less liquid into the tube a, so that here its content is near the critical state. The course of the temperature along

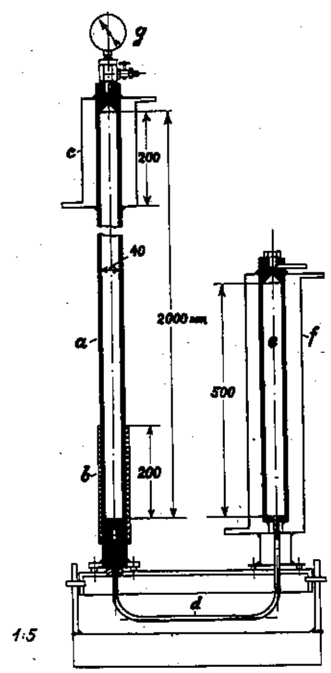

FIG. 19 – INSTALLATION FOR STUDYING HEAT TRANSFER IN VERTICAL TUBES AT CRITICAL STATE CONDITIONS
a) vertical tube
b) electrical heating
c) water cooling
d) connecting capillary
e) second tube for adjusting the state of the fluid
f) water jacket

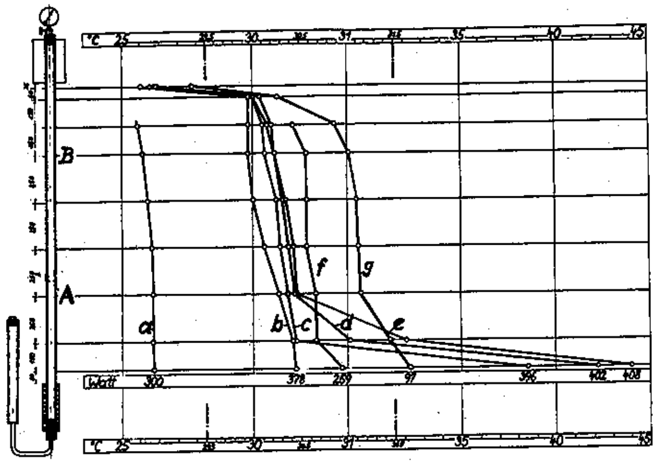

FIG. 20 – COURSES OF TEMPERATURE ALONG A TUBE FILLED WITH CARBON DIOXYDE NEAR ITS CRITICAL STATE

the tube *a* can be measured with the help of a number of thermocouples placed at distances given in Fig. 20, which shows at the left hand side the installation in a reduced scale. In the main part of the figure the curves *a* until *g* represent courses of temperature along the tube if it is filled with carbondioxide near its critical point. The temperatures can be read off from the scale above and below the picture, but it has to be noticed that between 29.5 and 31.5 C one degree corresponds to a length five times longer than below and above this range. At the lower end of each curve the heat flow in watts is noted. It is obvious to calculate from the heat flow (given by the electric energy of the heater or by the increase in temperature of the cooling water) and the gradient of temperature along the tube the apparent heat conductivity of the fluid including the transport of heat by natural convection. This is the heat conductivity which a solid body replacing the fluid should have in order to transport heat at the same rate as the fluid.

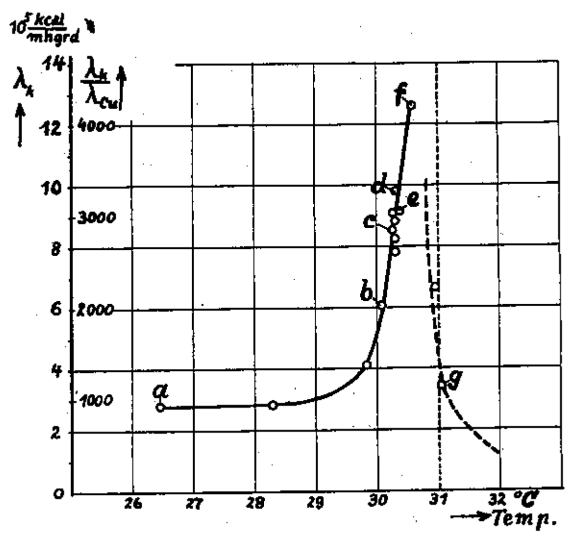

FIG. 21 – APPARENT OR EQUIVALENT HEAT CONDUCTIVITY OF CO_2 NEAR ITS CRITICAL STATE

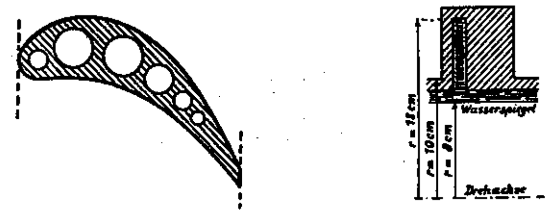

FIG. 22 – CROSS SECTION OF WATER COOLED GAS TURBINE BLADES

The result of these calculations is given in Fig. 21, where the apparent heat conductivity λ_k of the part AB in the middle of the tube and the fluid in it is plotted against its mean temperature. The figures at some of the points correspond to the curves in Fig. 20. For better illustration λ_k is not only given in kcal/mh deg but also as the ratio λ_k / λ_{Cu} to the heat conductivity $\lambda_{Cu} = 300$ kcal/mh deg of copper. The full line in Fig. 21 shows that towards the critical point the apparent heat conductivity of the fluid reaches values of 4000 times that of copper. But the exactness of this number should not be exaggerated because it is based on the measurements at very small differences of temperature.

d. Natural Convection at High Centrifugal Acceleration and Its Application for Cooling Gas Turbine Blades. High Rayleigh numbers are also possible in rotating bodies for instance in the rotors of turbines, where the centrifugal acceleration goes up to about 30,000 fold that of gravity and in ultracentrifuges even ten times larger accelerations are attainable.

The increase of natural convection by high centrifugal acceleration can be used for cooling blades of gas turbines following a proposal of E. Schmidt [31]. For that purpose the blades have a number of holes closed at the top of the blade and opening at its foot into the hollow drum of the turbine rotor, partly filled with water, as shown schematically in Fig. 22 a and b. Figure 23 is a photo of such a gas turbine rotor before closing the end of the holes by welding.

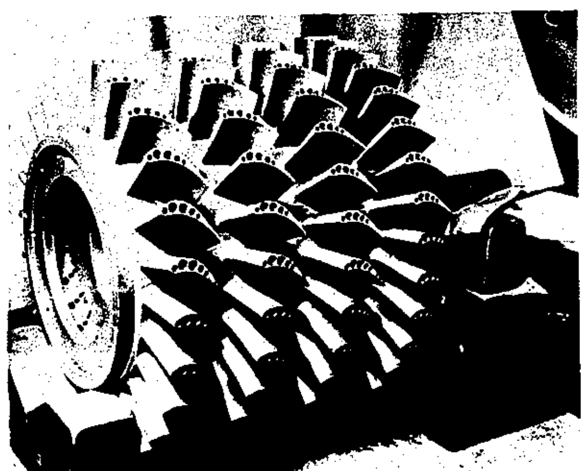

FIG. 23 – ROTOR OF A GAS TURBINE WITH COOLED BLADES BEFORE CLOSING BY WELDING THE HOLES IN THE BLADES

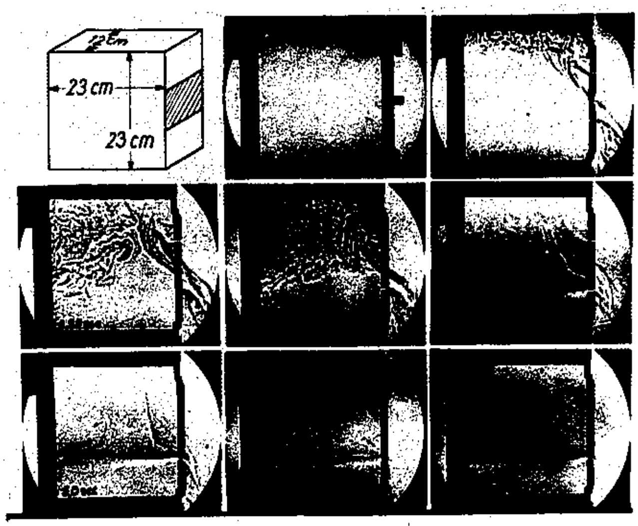

FIG. 24 – TRANSIENT EXCHANGE OF MATTER AND HEAT THROUGH THE OPENING OF A CONTAINER OR THE WINDOW OF A SITTING-ROOM

By rotation the water forms a ring in the rotor and also enters the holes of the blades. Due to the high centrifugal acceleration the water heated at the inner surface of the holes, undergoes an intensive natural convection which cools the blades very effectively. Because the pressure increases from the free surface of the water to the end of the blade, evaporation only takes place near the free surface. With an appropriate pressure of the steam in the drum it easily can be arranged, that the water in the holes nears the critical state so that natural convection is still increased as shown above. A gas turbine of about 3000 kW with this system for cooling the blades recently made a four hundred hours run with a gas temperature of 1050 C.

4. NATURAL CONVECTION AND MASS TRANSFER

The differences of density which produce natural convection can be brought about not only by temperature but also by fluids of different composition which mix with each other at uniform temperature by turbulence or by diffusion. In many cases a heat transfer experiment can be used to solve a problem of mixing and diffusion and vice versa.

As an example of this application the transient exchange of heat or matter through the opening of a container may be treated. In this way we can for instance answer the simple question: how long has the window of a sitting room to be opened at a certain difference of temperature inside and outside until the air in the room is renewed.

For this purpose a model container in form of a cube of the dimension given in the upper left corner of Fig. 24 was built. At its right side it has a window, noted by hatching which can be opened at time zero. The front and back walls of the container are made of optical glass in order to allow schlieren photographs through it. The closed container is filled with carbon dioxide, of the same temperature as the surrounding air. If the window is opened, CO_2 leaves the container through the lower half of the window and air enters through its upper half. The progress of this phenomenon is shown in eight photographs taken at different times noted on the photographs. We see that about 6 sec after opening the window the CO_2 in the upper two thirds of the container is replaced by air and only the lower third of it is still filled with CO_2. To remove this gas is a very slow process because it can only be accomplished by diffusion, if there is no forced convection in the container.

In translating the conditions of this experiment to a sitting room of about ten times greater linear dimensions, we find that the difference in density of CO_2 at the dimensions of the model corresponds to a temperature difference of about 10 C of the air inside and outside the sitting room. However, the time necessary for the exchange has to be enlarged by about the factor 100. Thus we have to open the window of our sitting-room for about 600 sec or ten minutes to get the same result of exchange as in the model apparatus.

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